Four Types of Sums of Squares for ANOVA Effects©

By default, SAS' PROC GLM gives one Type I and Type III *SS*. One may obtain nondefault types by specifying the type(s) desired in the MODEL statement. For example, "model rating=D|G|P / SS1 SS2 SS3 SS4;" would give all four types of *SS*.

Download the data file, SS1234.dat and the program SS1234.sas. Run the program and print the program and the listing so you will have examples of the types of *SS* presented here. The data for this exercise are from the research which was presented in the article: Castellow, W. A., Wuensch, K. L., & Moore, C. H. (1990). Effects of physical attractiveness of the plaintiff and defendant in sexual harassment judgments, *Journal of Social Behavior and Personality*, *5*, 547-562. The classification variables are DEATTR (experimentally manipulated physical attractiveness of the male defendant accused of sexual harassment), GENDER (gender of the mock juror), and PLATTR (experimentally manipulated physical attractiveness of the female plaintiff). I shall refer to the classification variables with the letters D, G, and P in the text below. The primary dependent variable in this research was the verdict recommended by the mock juror, but we also asked the jurors to rate the litigants on a number of characteristics. For this exercise, our dependent variable is one of those ratings, the jurors' ratings of the defendant's physical attractiveness.

Type I *SS*

Type I *SS* are order-dependent (hierarchical, **sequential**). Each effect is adjusted for all other effects that appear earlier (to the left) in the model, but not for any effects that appear later in the model. For example, in the first PROC GLM in "SS1234 SAS" the "rating=D|G|P" was expanded to "rating = D G D∗G P D∗P G∗P D∗G∗P" and the following adjustments were made:

Effect	Adjusted for
D	nothing
G	D
D∗G	D G
P	D G D∗G
D∗P	D G P D∗G
G∗P	D G P D∗G D∗P
D∗G∗P	D G P D∗G D∗P G∗P

If you will compare these Type I *SS* to those produced by the second PROC GLM, in which the order was P G G*P D D∗P D∗G D∗G∗P, you will see the order-dependent nature of Type I *SS*.

model rating =deattr|gender|plattr / ss1

Source	Type I SS
deattr	1291.887816
gender	4.531348
deattr*gender	15.873457
plattr	0.929137
deattr*plattr	0.140862
gender*plattr	0.666749
deattr*gender*plattr	4.251876

model rating = plattr|gender|deattr / ss1

Source	Type I SS
plattr	0.429308
gender	3.566900
gender*plattr	1.201155
deattr	1292.833415
deattr*plattr	0.034968
deattr*gender	15.963624
deattr*gender*plattr	4.251876

Type I *SS* are computed as the decrease in the Error *SS* (increase in the Model *SS*) when the effect is added to a model. For example, if *SSE* for Y=A is 100 and *SSE* for Y=A B is 80, then the Type I *SS* for B is 20. The sum of all of the effects' *SS* will equal the total Model *SS* for Type I *SS*—this is not generally true for the other types of *SS* (which exclude some or all of the variance that cannot be unambiguously allocated to one and only one effect). When cell sizes are equal, Types I, II, III, and IV are identical. In this case, Type I is preferred simply because it is computationally simpler.

This type of *SS* is also known as **Overall and Spiegel's** [1969, *Psychological Bulletin*, *72*: 311-322] **Method III**. Overall and Spiegel referred to an analysis using this type of *SS* as a "stepdown analysis."

Type I *SS* are appropriate for balanced (orthogonal, equal *n*) analyses of variance in which the effects are specified in proper order (main effects, then two-way interactions, then three-way interactions, etc.) and for trend analysis where the powers for the quantitative factor are ordered from lowest to highest in the model statement. Type I *SS* are also used for hierarchical stepdown nonorthogonal analyses of variance such as that advocated by Applebaum and Cramer [1974, *Psychological Bulletin. 81*: 335-343]. With such procedures one obtains the particular *SS* needed (adjusted for some effects but not for others) by carefully ordering the effects in the MODEL statement. If one can assign some effects priority (because they are temporally prior to other effects or because one's theory supposes they are causes of variance in other effects), e puts those effects to the left in the model statement.

Type II *SS*

Type II *SS* are the reduction in the *SSE* due to adding the effect to a model that contains all other effects except effects that contain the effect being tested. An effect is contained in another effect if it can be derived by deleting terms in that effect—for example, A, B, C, A∗B, A∗C, & B∗C are all contained in A∗B∗C

David Howell has described the hypotheses tested by Type II sums of squares as "peculiar" and "very bizarre" (page 595 of *Statistical Methods for Psychology*, 7th ed.). Type II sums of squares do, however, have their advocates (see Donald Macnaughton's paper).

The Type II *SS* for our example involve the following adjustments (regardless of the order of the effects in the model statement).

Effect	Adjusted for
D	G P G∗P
G	D P D∗P
D∗G	D G P D∗P G∗P
P	D G D∗G
D∗P	D G P D∗G G∗P
G∗P	D G P D∗G D∗P
D∗G∗P	D G P D∗G D∗P G∗P

model rating =deattr|gender|plattr / ss2

Source	Type II SS
deattr	1292.833415
gender	4.720618
deattr*gender	15.963624
plattr	0.929137
deattr*plattr	0.145936
gender*plattr	0.666749
deattr*gender*plattr	4.251876

model rating = plattr|gender|deattr / ss2

Source	Type II SS
plattr	0.929137
gender	4.720618
gender*plattr	0.666749
deattr	1292.833415
deattr*plattr	0.145936
deattr*gender	15.963624
deattr*gender*plattr	4.251876

Types III & IV *SS*

Type III *SS* are each adjusted for all other effects in the model, regardless of order. GLM'S Type III *SS* are also known as **Overall and Spiegel's Method I** *SS* ("complete linear model analysis"). Howell and McConaughy [1982, *Educ. & Psychol. Measurement: 42*, 9-24] strongly recommend use of these *SS* in nonorthogonal ANOVA. These are also the *SS* that are approximated by the traditional "**unweighted means**" ANOVA that uses harmonic mean sample sizes to adjust cell totals.

model rating =deattr|gender|plattr / ss3

Source	Type III SS
deattr	1275.997766
gender	4.068253
deattr*gender	15.893562
plattr	0.836559
deattr*plattr	0.180523
gender*plattr	0.791002
deattr*gender*plattr	4.251876

model rating = plattr|gender|deattr / ss3

Source	Type III SS
plattr	0.836559
gender	4.068253
gender*plattr	0.791002
deattr	1275.997766
deattr*plattr	0.180523
deattr*gender	15.893562
deattr*gender*plattr	4.251876

Type IV *SS* are identical to Type III *SS* for designs with no missing cells, and when you have missing cells you really should not be doing a factorial analysis. Although you can do one with Type IV *SS*, the solution is not unique -- better to collapse across or eliminate categories in order to avoid the missing cells.

Least Squares Means

LSMEANS provides you with estimates of what the means would be if the independent variables were not correlated with one another. I requested LSMEANS only for the deattr effect, which was the only significant effect. If you look at those means, you will see that our manipulation of the physical attractiveness of the defendant was quite effective.

deattr	rating LSMEAN
No	1.81871883
Yes	7.76425439

SPSS for Windows

If you are using SPSS for Windows, you can also get four types of sums of squares, as you will see when you read my document Three-Way Nonorthogonal ANOVA on SPSS.

PROC REG

For multiple regressions using SAS' PROC REG, Type I *SS* are sequential *SS* (each effect adjusted only for effects that precede it in the model) and Type II *SS* are unique *SS* (each effect is adjusted for all other effects in the model). Likewise, PCORR1 and SCORR1 are squared sequential partial and semi-partial correlation coefficients, PCORR2 and SCORR2 adjust for all terms in the model.

Return to Karl's SAS Lessons Page