

ECONOMICS 3343: BRIEF INTRODUCTION TO THE CAPM AND FACTOR MODELS

PART I: THE CAPITAL ASSET PRICING MODEL

This is a short introduction to the Capital Asset Pricing Model (CAPM). Under a certain **set of assumptions**, **William Sharpe** showed that the following relationship holds, for each observation t , between the expected rate of return on a **security**, and a linear combination of the **risk-free rate of return** and expected rate of return on the **market portfolio**:

$$E(R_t) = R_{ft} + \beta_M(E(R_{Mt}) - R_{ft}), \quad (1)$$

where:

$E(R_t)$ is the expected rate of return on the security

R_{ft} is the risk-free rate of return

$E(R_{Mt})$ is the expected rate of return on the market portfolio

β_M is the “beta” for the security

The parameter beta is a **measure of risk** for the security. We are often interested in beta being equal to 1, less than 1, or greater than 1. As explained **here**, if beta is greater than 1, the security’s returns are more volatile than the market’s rate of return; if it’s less than 1, its returns are less volatile than the market’s; and if it’s equal to 1, its returns are just as volatile as the market’s rate of return.

The CAPM is assumed to hold for each observation t . The CAPM relationship can be used to compute the expected rate of return on a security, given values for: (a) the risk-free rate of return; (b) the expected rate of return on the market portfolio; and (c) the value of beta for the security. Note that if we subtract the risk-free rate of return from both sides of the equation (1), we get:

$$E(R_t) - R_{ft} = \beta_M(E(R_{Mt}) - R_{ft}). \quad (2)$$

The variable on the left side of equation (2) is the “**risk premium**” for the security and, on the right side of equation (2), this security’s beta is multiplied by the “**risk premium on the market portfolio**.” Equation (2) is the basis of the following time-series regression model of the CAPM:

$$R_t - R_{ft} = \alpha + \beta_M(R_{Mt} - R_{ft}) + \epsilon_t, \quad (3)$$

where α is the “alpha” for the security and a stochastic error term, ϵ_t , has been added. The parameter alpha is called “**Jensen’s alpha**,” after **Michael Jensen**, who noted **in this paper** that if the CAPM holds, then the expected value of alpha for the security in the regression given by equation (3) is zero. More generally, the alpha for a security is a risk-adjusted measure of the rate of return obtained from investing in the security. Both the dependent and independent variables in Equation (3) are in the form of an “excess return,” where the “excess” of the return is measured in terms of the difference from the risk-free rate of return. Note: A variant of equation (3) appears on p. 32 of **this excellent paper** on the CAPM, which provides a far more thorough discussion of the derivation of the time-series regression model given in equation (3); the only difference is that the subscript i is suppressed here, in order to simplify the notation.

You should recognize equation (3) as a “simple regression model” (in which the dependent variable is the excess return for the security and the independent variable is the excess return for the market portfolio); the intercept is the alpha for the security and the slope is the beta for the security. Accordingly, if equation (3) is estimated by OLS, you should see how $\hat{\beta}_M$ is computed and that should help you understand the interpretation of beta as a measure of risk for the security, i.e., it measures how much the rate of return on the security “covaries” with the rate of return of the market portfolio, relative to the variance of the market portfolio’s rate of return.

PART II: FACTOR MODELS

For our purposes, we’ll consider the model known as Arbitrage Pricing Theory (APT) to be an extension of the CAPM. The APT was developed by **Stephen Ross**, who showed that under a particular set of assumptions (which are discussed [here](#)), the following relationship holds for the rate of return of a security:

$$R_t = R_{ft} + \beta_1 F_{1t} + \beta_2 F_{2t} + \cdots + \beta_K F_{Kt} + \epsilon_t, \quad (4)$$

where F_{jt} = “systematic factor” j , $j = 1, \dots, K$. The term “systematic factor” means a key variable which influences investment risk. The rate of return for the security given in equation (4) is said to have a “factor structure.” If R_{ft} is subtracted from both sides of equation (4), and the alpha for the security, α , which should be zero as noted above, is added to the right side, we get:

$$R_t - R_{ft} = \alpha + \beta_1 F_{1t} + \beta_2 F_{2t} + \cdots + \beta_K F_{Kt} + \epsilon_t, \quad (5)$$

Note that if $K=1$ and $F_{1t} = R_{Mt} - R_{ft}$, then equation (5) reduces to equation (3); in this case $\beta_1 = \beta_M$. So, the CAPM can be considered a single-factor factor model, with that single factor being the excess return on the market portfolio.

In [this 1986 paper](#), Chen, Roll, and Ross investigated whether a group of macroeconomic factors, when added to the basic CAPM, helped explain the movement in the rates of return to investing in a wide set of U.S. stocks. One such macroeconomic factor of interest for the course paper is what they called the “unanticipated change in the risk premium.” As in [this paper](#), we’ll measure this variable by the difference between the yields on Baa and Aaa rated bonds. We’ll use the notation “*CRP*” (for change in risk premium) to refer to this difference in the Baa and Aaa bond yields; in the literature this is also called the “default premium” and “change in the default risk premium.” When this variable is added to the time-series regression model for estimating the simple CAPM, we get:

$$R_t - R_{ft} = \alpha + \beta_M(R_{Mt} - R_{ft}) + \beta_c CRP_t + \epsilon_t, \quad (6)$$

In [this 1993 paper](#), **Fama** and **French** proposed a three-factor model for the returns on a security:

$$R_t - R_{ft} = \alpha + \beta_M(R_{Mt} - R_{ft}) + \beta_s SMB_t + \beta_h HML_t + \epsilon_t, \quad (7)$$

where:

SMB_t (for “small minus big”) is the difference between the returns on diversified portfolios of small and big stocks, where “small” and “big” are determined by the size of market capitalization (price times the number of shares outstanding) on the underlying stocks.

SMB_t is also called the “size premium.” Additional discussion about this factor can be found “[here](#).”

HML_t (for “high minus low”) is difference between the returns on diversified portfolios of high and low “book-to-market equity ratios”, where book-to-market equity ratio means ratio of the “book value” of a common stock to its market value, and the notion of book value is explained [here](#). HML_t is also called the “value premium.” Additional discussion about this factor can be found “[here](#).”

The Fama and French three-factor model is used, along with the simple CAPM, ubiquitously in the modeling of excess returns on various financial assets, both in the academic literature and in practice by financial firms. More details about the Fama and French three-factor model can be found on pp. 37-39 of [this paper](#).

If we add the factor CRP to the Fama and French three-factor model in equation (7), we get:

$$R_t - R_{ft} = \alpha + \beta_M(R_{Mt} - R_{ft}) + \beta_c CRP_t + \beta_s SMB_t + \beta_h HML_t + \epsilon_t. \quad (8)$$

We’ll refer to the four-factor model given by equation (8) as the “combined model.”

Some Hypothesis Tests Which Are Relevant for the Course Paper:

- I. $H_0: \alpha = 0$ against $H_A: \alpha \neq 0$.
- II. $H_0: \beta_M = 0$ against $H_A: \beta_M \neq 0$.
- III. $H_0: \beta_M = 1$ against $H_A: \beta_M \neq 1$. If this null is rejected, it’s important to note if $\hat{\beta}_M$ is significantly below or above 1.
- IV. $H_0: \beta_c = 0$ against $H_A: \beta_c \neq 0$.
- V. $H_0: \beta_s = 0$ against $H_A: \beta_s \neq 0$.
- VI. $H_0: \beta_h = 0$ against $H_A: \beta_h \neq 0$.

Note: You’ll be asked to consider some other hypothesis tests for the course paper.