

Calculation of grain-size statistics and parameters

Statistics of the grain-size distribution were computer calculated with a U.S. Geological Survey Marine Geology grain-size program (McHendrie 1988). Grain-size statistical parameters and graphic representations are given in phi units. The phi unit (Φ) is a logarithmic transformation of millimeters into whole integers, according to the formula: $\Phi = -\log_2 d$, where d = grain diameter in millimeters.

The parameters calculated for these analyses include:

1. "median" - corresponds to the 50 percentile on a cumulative curve, where half the particles by weight are larger and half are smaller than the median. This parameter is measured in phi units.
2. "mean" - is the average grain-size. Several formulas are used in calculating the mean. The most inclusive graphically derived value is that given by Folk (1968):

$M_z = \frac{\Phi_{16} + \Phi_{50} + \Phi_{84}}{3}$, where Φ_{16} , Φ_{50} , and Φ_{84} represent the size at 16, 50, and 84 percent of the sample by weight. Mean is also measured in phi units and is the most widely compared parameter.

3. "sorting" - is a method of measuring the grain-size variation of a sample by encompassing the largest parts of the size distribution as measured from a cumulative curve. Folk (1968) introduced the "inclusive graphic standard deviation", that is calculated as follows:

$\sigma_1 = \frac{\Phi_{84} - \Phi_{16}}{4} + \frac{\Phi_{95} - \Phi_5}{6.6}$, where Φ_{84} , Φ_{16} , Φ_{95} , and Φ_5 represent the phi values at 84, 16, 95, and 5 percentiles. Folk (1968) presented a verbal classification scale for sorting: $\sigma_1 < 0.350$: very well sorted; 0.35-0.500: well sorted; 0.5-0.710: moderately well sorted; 0.71-1.00: moderately sorted; 1.00-2.00: poorly sorted; 2.00-4.00: very poorly sorted; and, >4.00 : extremely poorly sorted.

4. "skewness" - measures the degree to which a cumulative curve approaches symmetry. Two samples may have the same average grain size and sorting but may be quite different to their degrees of symmetry. Folk's "inclusive graphic skewness" (1968) is determined by the equation:

$sk_1 = \frac{\Phi_{16} + \Phi_{84} - 2\Phi_{50}}{2(\Phi_{84} - \Phi_{16})} + \frac{\Phi_5 + \Phi_{95} - 2\Phi_{50}}{2(\Phi_{95} - \Phi_5)}$, where the phi values represent the same percentages as those for sorting. This formula includes a measure of the "tails" of the cumulative curve as well as the central portion. Other methods for determining skewness, notably those by Inman (1952) and Trask (1950), do not measure the tails of the curve. Symmetrical curves have a skewness equal to 0.00; those with a large proportion of fine material are positively skewed; those with a large proportion of coarse material are negatively skewed. A verbal classification for skewness suggested by Folk (1968) includes: sk_1 from +0.10 to -0.10 as nearly symmetrical; -0.10 to -0.30 as coarse-skewed; and, -0.30 to -1.00 as strongly coarse-skewed.

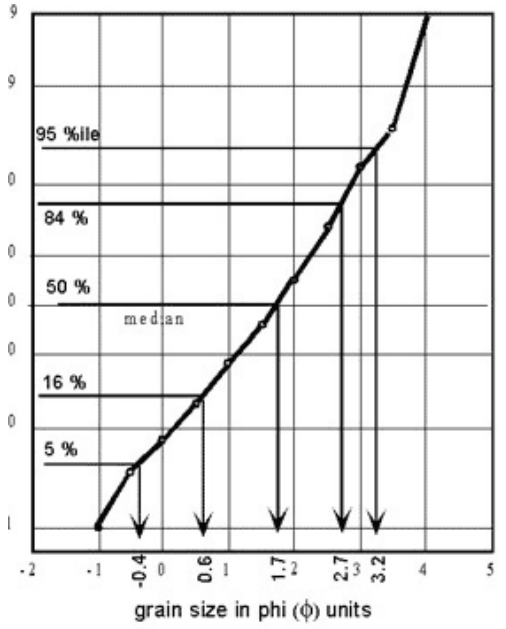
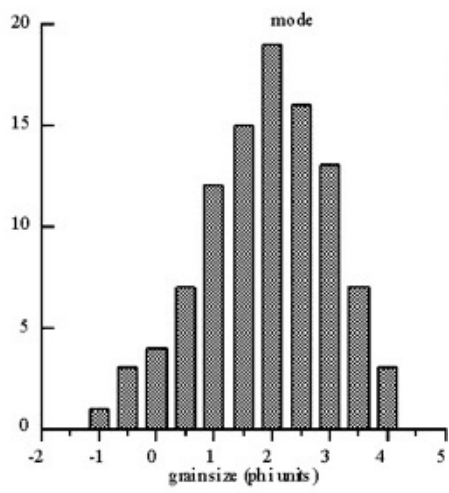
5. "kurtosis" - is a measure of "peakedness" in a curve. Folk's (1968) formula for kurtosis is:

$k_g = \frac{\Phi_{95} - \Phi_5}{2.44(\Phi_{75} - \Phi_{25})}$, where the phi values represent the same percentages as those for sorting. A normal Gaussian distribution has a kurtosis of 1.00 which is a curve with the sorting in the tails equal to the sorting in the central portion. If a sample curve is better sorted in the central part than in the tails, the curve is said to be excessively peaked, or leptokurtic; if the sample curve is better sorted in the tails than in the central portion, the curve is flat peaked or platykurtic. For normal curves $k_g = 1.00$, leptokurtic curves have $k_g > 1.00$, and platykurtic curves have $k_g < 1.00$.

		US Standard sieve mesh	Millimeters	Phi (ϕ) units	Wentworth size class	
GRAVEL			4096	-12		
			1024	-10	Boulder	
			256	256	-8	
			64	64	-6	Cobble
			16		-4	
		5	4	4	-2	Pebble
		6	3.36		-1.75	
		7	2.83		-1.5	Granule
		8	2.38		-1.25	
		10	2.00	2	-1.0	
SAND		12	1.68		-0.75	
		14	1.41		-0.5	Very coarse sand
		16	1.19		-0.25	
		18	1.00	1	0.0	
		20	0.84		0.25	
		25	0.71		0.5	Coarse sand
		30	0.59		0.75	
		35	0.50	1/2	1.0	
		40	0.42		1.25	
		45	0.35		1.5	Medium sand
		50	0.30		1.75	
		60	0.25	1/4	2.0	
		70	0.210		2.25	
		80	0.177		2.5	Fine sand
		100	0.149		2.75	
		120	0.125	1/6	3.0	
		140	0.105		3.25	
		170	0.088		3.5	Very fine sand
		200	0.074		3.75	
	MUD	SILT	230	0.0625	1/16	4.0
270			0.053		4.25	
325			0.044		4.5	Coarse silt
			0.037		4.75	
			0.031	1/32	5.0	
CLAY			0.0156	1/64	6.0	Medium silt
			0.0078	1/128	7.0	Fine silt
			0.0039	1/256	8.0	Very fine silt
			0.0020		9.0	
			0.00098		10.0	Clay
	0.00049		11.0			
	0.00024		12.0			
	0.00012		13.0			
	0.00006		14.0			

size (phi)	weight	weight percent	cumulative frequency
-1.0	1.0	1	1
-0.5	2.5	3	4 (1+3)
0.0	3.3	4	8 (4+4) etc.
0.5	5.8	7	15
1.0	10.0	12	27
1.5	12.3	15	42
2.0	15.2	19	61
2.5	13.3	16	80
3.0	10.2	13	93
3.5	5.5	7	97
4.0	2.1	3	100
<4	0.3	<1	
	81.5		
	total weight		(10.2/81.5) = 13%

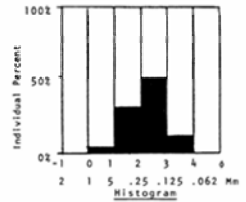
data



Graphical Formulas:

Mode

Mode (Mo) is the most frequently-occurring particle diameter.



Median

Median (Md). Half of the particles by weight are coarser than the median, and half are finer. It is the diameter corresponding to the 50% mark on the cumulative curve

Graphic Mean

$$M_z = (\phi_{16} + \phi_{50} + \phi_{84})/3$$

Inclusive Graphic Standard Deviation (sorting)

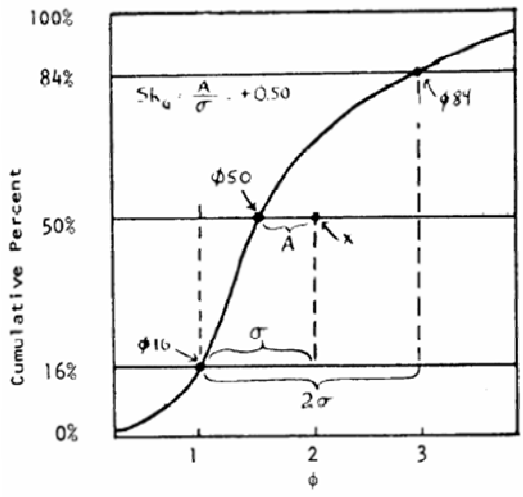
$$\frac{\phi_{84} - \phi_{16}}{4} + \frac{\phi_{95} - \phi_5}{6.6}$$

Inclusive Graphic Skewness

$$\frac{\phi_{16} + \phi_{84} - 2\phi_{50}}{2(\phi_{84} - \phi_{16})} + \frac{\phi_5 + \phi_{95} - 2\phi_{50}}{2(\phi_{95} - \phi_5)}$$

Inclusive Graphic Kurtosis

$$K_G = \frac{\phi_{95} - \phi_5}{2.44 (\phi_{75} - \phi_{25})}$$



Sorting

Values from	To	Equal
0.00	0.35 ϕ	very well sorted
0.35	0.50 ϕ	well sorted
0.50	0.71 ϕ	moderately well sorted
0.71	1.00 ϕ	moderately sorted
1.00	2.00 ϕ	poorly sorted
2.00	4.00 ϕ	very poorly sorted
4.00	ϕ	extremely poorly sorted

Skewness

Values from	To	Mathematically:	Graphically Skewed to the:
+1.00	+0.30	Strongly positive skewed	Very Negative phi values, coarse
+0.30	+0.10	Positive skewed	Negative phi values
+0.10	- 0.10	Near symmetrical	Symmetrical
- 0.10	- 0.30	Negative skewed	Positive phi values
- 0.30	- 1.00	Strongly negative skewed	Very Positive phi values, fine

Kurtosis

Values from	To	Equal
0.41	0.67	very platykurtic
0.67	0.90	platykurtic
0.90	1.11	mesokurtic
1.10	1.50	leptokurtic
1.50	3.00	very leptokurtic
3.00		extremely leptokurtic

Method of Moments

All of the above statistical parameters can be calculated using the method of moments. This method gives a more rigorous treatment of the sediment characteristics. The first moment measure corresponds to the mean, the second to the standard deviation, the third to the skewness, and the fourth to the kurtosis. (from U.S. Geological Survey Open-File Report 91-375-A)

The term moment was introduced into statistics by analogy with mechanics. In mechanics, the moment of a force about a point of rotation, such as a fulcrum, is computed by multiplying the magnitude of that force by the distance from the force to the point of rotation. In statistical moments the force of mechanics is replaced by a frequency function (such as the percentage of the distribution within a given class interval). The concept of the distance to a point is the same in both kinds of moments. In statistical moments the point of rotation of mechanics is replaced by an arbitrary point, commonly either the origin of the curve or its mean. The distance is determined by the spacing of the size class from the arbitrary point.

The statistical moment is the moment of a given size class with respect to the arbitrary point (i.e., to the origin of the curve). The statistical moment of the distribution is the moment per unit frequency. It is determined by finding the moments of each size class (frequency percentage in each size class times the distance of each from the origin), adding them up, and dividing the sum by 100.

The first moment is the mean: $\bar{x} = \frac{\sum fm_{\phi}}{100}$ where f is frequency in percent for each size class and m_{ϕ} is the midpoint of each ϕ class. In the first moment the distance term (m_{ϕ}) appears in the first power. Successively higher moments are defined by raising the distance term to progressively higher powers. In the higher moments the distance term is represented with respect to the mean, so the higher moments are moments about the mean, whereas the first moment is the moment with respect to an arbitrary point. The second moment is the square of the standard deviation. The third moment is the skewness, and the fourth moment is the kurtosis. (from <http://www.log.furg.br/WEBens/morelock/moments.htm>)

Method of Moments Formulas:

$$\text{Mean} = \bar{x} = \frac{\sum(fm)}{\sum(f)}$$

$$\text{Standard deviation} = \sigma = \sqrt{\frac{\sum f(m - \bar{x})^2}{100}}$$

$$\text{Skewness} = \frac{\sum f(m - \bar{x})^3}{100 * \sigma^3}$$

$$\text{Kurtosis} = \frac{\sum f(m - \bar{x})^4}{100 * \sigma^4}$$

Mode = most frequently occurring grain-size

Median = grain size for which 50% of the grains in the sample are larger and 50% are smaller.