Some Formulas to Keep in Mind for the Final Exam

1. The formula relating the real rate of return to the nominal interest rate of interest and the rate of inflation, with annual compounding, is:

   \[
   \text{Real Rate} = \frac{\text{Nominal Interest Rate} - \text{Inflation Rate}}{1 + \text{Inflation Rate}}.
   \]

2. For 'single cash flows', the relationship between \( FV \) and \( PV \), given \( i \) (expressed as a decimal fraction) and \( n \), is:

   \[
   FV = PV \times (1 + i)^n \iff PV = \frac{FV}{(1+i)^n}.
   \]

3. Given an \( APR \), the \( EFF \) is found by:

   \[
   EFF = (1 + \frac{APR}{m})^m - 1, \quad \text{where} \quad m \text{ is the number of compounding periods.}
   \]

4. For an ordinary annuity of $1 per period for \( n \) periods at an interest \( i \) (expressed as a decimal fraction), the formula for \( FV \) is:

   \[
   FV = \frac{(1+i)^n - 1}{i}.
   \]

5. For an ordinary annuity of $1 per period for \( n \) periods at an interest rate \( i \) (expressed as a decimal fraction), the formula for \( PV \) is:

   \[
   PV = \frac{1 - (1+i)^{-n}}{i}.
   \]

6. The formula for the \( PV \) of a level perpetuity is:

   \[
   PV \text{ of a Level Perpetuity} = \frac{C}{i}, \quad \text{where} \quad C \text{ is the periodic payment and } i \text{ is the interest rate (expressed as a decimal fraction).}
   \]

7. The yield to maturity for a coupon bond is the discount rate (i.e., the value of \( i \) expressed as a decimal fraction) that solves the following equation:

   \[
   \text{Bond Price} = \sum_{t=1}^{n} \frac{\text{Coupon Payment}}{(1+i)^t} + \frac{\text{Face Value}}{(1+i)^n},
   \]

   where \( n \) is the number of annual payment periods until the bond’s maturity.
8. The DDM implies:

\[ P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}, \]

where \( P_0 \) is the current price of a share of stock, \( D_t \) is the expected dividend payment in period \( t \), and \( k \) is the market capitalization rate.

9. The Constant-Growth-Rate DDM implies:

\[ P_0 = \frac{D_1}{k-g}, \]

where \( P_0 \) is the current price of a share of stock, \( D_1 \) is next period’s expected dividend payment, \( k \) is the market capitalization rate, and \( g \) is the expected constant growth rate of dividend payments.

10. The formula for the growth rate of dividends and earnings per share is:

\[ g = \text{Earnings Retention Rate} \times \text{Rate of Return on New Investments} \]

11. The value of a share of a firm’s stock can be estimated by:

\[ P_0 = \frac{E_1}{k} + \text{Net Present Value of Future Investment Opportunities}, \]

where \( P_0 \) is the current price of a share of stock, \( E_1 \) is next period’s expected earnings, and \( k \) is the market capitalization rate.

12. If \( E(r_s) \) is the expected return on the risky asset, \( r_f \) is the expected return on the riskless asset, and a portfolio is constructed by placing a weight \( w \) on the risky asset and a weight \( (1-w) \) on the risk-free asset, then the expected return on the portfolio is given by:

\[ E(r) = wE(r_s) + (1-w)r_f. \]

13. Consider a portfolio constructed by placing the weight \( w \) on a risky asset and the weight \( (1-w) \) on the riskless asset. If \( \sigma_s \) is the standard deviation of the risky asset, then the standard deviation of the portfolio is given by:

\[ \sigma = \sigma_s w. \]

14. If \( E(r_s) \) is the expected return on the risky asset and \( \sigma_s \) is the standard deviation of the risky asset, the equation for the Risk-Reward Trade-Off line is:

\[ E(r) = r_f + \frac{E(r_s)-r_f}{\sigma_s} \sigma. \]

When graphed in the \((\sigma, E(r))\) plane, the slope of the Risk-Reward Trade-Off line is given by \( \frac{E(r_s)-r_f}{\sigma_s} \), which is also known as the “Reward-to-Risk” ratio.

15. Note that Points #13 and #14 above also imply that along the Risk-Reward Trade-Off line:

\[ E(r) = r_f + (E(r_s) - r_f)w. \]

This can also be considered an equation for the Risk-Reward Trade-Off line; to see this recall what \( w \) equals.
16. If $E(r_1)$ and $E(r_2)$ are the expected returns for Risky Asset 1 and Risky Asset 2, respectively, then the expected return on any portfolio consisting of a proportion $w$ in Risky Asset 1 and a proportion $(1 - w)$ is Risky Asset 2 is given by:

$$E(r) = wE(r_1) + (1 - w)E(r_2).$$

17. If $\sigma_1^2$ and $\sigma_2^2$ are the variances for Risky Asset 1 and Risky Asset 2, respectively, then the variance of any portfolio consisting of a proportion $w$ in Risky Asset 1 and a proportion $(1 - w)$ in Risky Asset 2 is given by:

$$\sigma^2 = w^2\sigma_1^2 + (1 - w)^2\sigma_2^2 + 2w(1 - w)\rho\sigma_1\sigma_2,$$

where $\rho$ is the correlation coefficient between returns on the two risky assets. Note that if “Risky Asset 2” is actually the riskless asset, then this variance expression reduces (upon taking the square root) to the relationship given in Point #13 above.

18. The formula for the proportion of the portfolio in Risky Asset 1 that minimizes the variance of the portfolio formed by combining Risky Asset 1 and Risky Asset 2 is given by:

$$w_{\text{min}} = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}.$$

19. If $T$ is the “Tangency Portfolio,” then the equation for the Risk-Reward Trade-Off line formed by combining the risk-free asset with the Tangency Portfolio is given by:

$$E(r) = r_f + \frac{[E(r_T) - r_f]}{\sigma_T}\sigma.$$

20. If $M$ is the “Market Portfolio” (with expected return equal to $E(r_M)$ and variance equal to $\sigma_M^2$), then the equation for the Capital Market Line is given by:

$$E(r) = r_f + \frac{[E(r_M) - r_f]}{\sigma_M}\sigma.$$

21. If $M$ is the Market Portfolio and $A$ is an index of the degree of risk aversion in the economy, then the CAPM implies that the following relationship holds in equilibrium:

$$E(r_M) - r_f = A\sigma_M^2.$$

22. The CAPM Security Market Line is given by:

$$E(r_j) - r_f = \beta_j[E(r_M) - r_f],$$

where

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2}.$$

23. If $w$ is the fraction of a portfolio on the Capital Market Line invested in the Market Portfolio, then the $\beta$ for this portfolio equals $w$. 

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