Solutions to Problems on Final Exam

1. Given $i = 0.06$ and $n = 7$, we apply the ordinary annuity $PV$ formula to compute this fixed-income security's price as follows:

   $\text{Bond Price} = \frac{1 - (1.06)^{-7}}{0.06} \times 125 = 697.78$

2. Given the yield curve information, the $PV$ of the three $140$ payments is given by:

   $PV = 0.96 \times 140 + 0.92 \times 140 + 0.87 \times 140 = 140 \times (0.96 + 0.92 + 0.87) = 385.0$

   This $PV$ is the price of the fixed-income security.

3. If $i = 8\%$, the price of the bond is given by:

   $\text{Bond Price} = \frac{1 - (1.08)^{-4}}{0.08} \times 110 = 364.33$

   If $i = 3\%$, the price of the bond is given by:

   $\text{Bond Price} = \frac{1 - (1.03)^{-4}}{0.03} \times 110 = 408.88$

   So, as $i$ goes from $8\%$ to $3\%$, the price of the bond changes from $364.33$ to $408.88$, creating a change in market value equal to:

   $364.33 - 408.88 = 44.55$

4. Straightforward calculations show that the yields to maturity for these zero-coupon bond prices are as follows:

   \[
   \begin{array}{|c|c|c|}
   \hline
   \text{Maturity} & \text{Compute } i: & \text{YTM} \\
   \hline
   1 \text{ year} & \frac{1}{(1+i)} & 0.93458 \quad 7\% \\
   2 \text{ years} & \frac{1}{(1+i)^2} & 0.89000 \quad 6\% \\
   3 \text{ years} & \frac{1}{(1+i)^3} & 0.86384 \quad 5\% \\
   4 \text{ years} & \frac{1}{(1+i)^4} & 0.85480 \quad 4\% \\
   \hline
   \end{array}
   \]

   So, the yield curve is downward-sloping for the range of maturities depicted.

5. We have $i = 0.04$, $n = 23$, $FV = 780$, and the price of the bond is given by the $PV$ of this $FV$ to be received 23 years from now. Since this is a pure discount bond, the interest rate on it is the appropriate interest rate to use as the discount rate. So, the price of the bond is given by:

   $\text{Bond Price} = \frac{780}{(1.04)^{23}} = 316.47$

6. Proceeding as in the solution for Problem #5, we still have $i = 0.04$ and $FV = 780$, but now $n = 13$. So, the price of the bond is given by:
Bond Price = $780 / (1.04)^{13} = $468.05

7. Given the solutions to Problems #5 and #6, we need to solve for \( g \) in the following:

\[
468.05 = (1 + g)^{10} \times 316.47 \Rightarrow (1 + g) = \left(\frac{468.05}{316.47}\right)^{\frac{1}{10}} \Rightarrow g = 0.03991
\]

8. Recall that:

\[
\text{Current Yield} = \frac{\text{Coupon Payment}}{\text{Bond Price}}.
\]

To compute this, we need to know the Coupon Payment. Since the face value is $1200 and the coupon rate is 6\%, the coupon payment is $72 \ ($1200 \times 0.06). So:

\[
\text{Current Yield} = \frac{72}{975} = 0.0738 \text{ (or 7.38\%)}
\]

9. Given that the coupon rate is 11\%, the face value is $1,050, and the current price of the bond is $825, the yield to maturity is the value of \( i \) that solves the following:

\[
825 = \frac{1.050 \times (1.11)}{1 + i} = \frac{1.165.5}{1 + i} \Rightarrow i = \frac{1.165}{825} - 1 = 0.41212 \text{ (or 41.212\%)}
\]

10. First we need to find the price of the bond eight months ago. It’s given by the following \( PV \) calculation, using \( n = 20, i = 0.02, \) and \( FV = $1,500: \)

\[
PV = \frac{1.500}{(1.02)^{20}} = $1009.46
\]

Next we compute the price of the bond today, using \( n = 19.25 \) (since three-quarters of a year have passed), \( i = 0.01 \) and keeping \( FV \) unchanged:

\[
PV = \frac{1.500}{(1.01)^{19.25}} = $1,238.53
\]

Then the rate of return is computed as:

\[
\text{Rate of return} = \frac{\text{coupon} + \text{change in bond price}}{\text{initial bond price}}
\]

Since this is a zero-coupon bond, the coupon payment equals 0, so we have:

\[
\text{Rate of return} = \frac{1.238.53 - 1.009.46}{1.009.46} = \frac{229.07}{1.009.46} = 0.22692 \text{ (or 22.692\%)}
\]

11. We’re given that \( D_1 = $7.50, k = 0.20, \) and \( P_0 = $15. \) So, according to the constant- growth-rate DDM, the expected growth rate of dividends is the value of \( g \) that solves the following:

\[
15 = \frac{7.5}{0.20 - g} \Rightarrow g = 0.20 - \frac{7.5}{15} = -0.3 \text{ (or -30.0\%)}
\]

12. We’re given that \( D_0 = $5, g = 0.10, \) and \( k = 0.12. \) So, according to the constant- growth-rate DDM, the imputed value of the stock \( (P_0) \) is given by:

\[
P_0 = \frac{5 \times (1 + 0.10)}{0.12 - 0.10} = \frac{5.5}{0.02} = $275.00
\]

13. We still have \( D_0 = $5 \) and \( g = 0.10. \) But now we’re told that \( P_0 = $110.00 \) and we need to solve for \( k. \) The solution is given by:
110 = \frac{5 \times (1 + 0.10)}{k - 0.10} \Rightarrow k = \frac{5.5}{110} + 0.10 = 0.15 \text{ (or 15%)}

14. By definition, we have:

\[ g = \text{earnings retention rate} \times \text{rate of return on new investments} = 0.65 \times 0.30 = 0.195 \text{ (or 19.5%)} \]

15. From Problem #14 we know that \( E_1 = \$4 \) and the earnings retention rate is 65%. This implies that:

\[ D_1 = 0.35 \times \$4 = \$1.40, \text{ which, along with other information given in Problem #14, implies:} \]

\[ P_0 = \frac{\$1.40}{25 - 0.195} = \$25.45 \]

16. The stock price (whose current value was found, in Problem #14, to be \$25.45) is expected to grow at the same rate as dividends (which, in Problem #13 was found to be 19.5% per year). So:

\[ P_1 = P_0 \times (1 + g) = \$25.45 \times 1.195 = \$30.41 \]

17. According to the constant-growth-rate DDM:

\[ P_{11} = \frac{D_{12}}{k - g} = \frac{14.25}{0.10 - 0.06} = \$356.25 \]

18. In Problem #17, we found that \( P_{11} = \$356.25 \). We now use the market capitalization rate \( k = 10\% \) to get the present value of \$356.25 received 11 years from now:

\[ P_0 = \frac{356.25}{(1.10)^{11}} = \$124.86 \]

19. If \( P_0 = \$65.25 \), then, given that the market capitalization rate is 10%, \( P_{11} = (1.10)^{11} \times \$65.25 = \$186.17 \). Next, we need to find the value of \( g \) that solves the following:

\[ \frac{186.17}{\$14.25 \over 0.10 - g} \Rightarrow g = 0.10 - \frac{14.25}{186.17} = 0.023457 \text{ (2.3457%)} \]

Note: Before the stock pays out dividends, its price is expected to grow at the market capitalization rate \( k \). Once it starts paying out dividends, its price is expected to grow at the expected growth rate of dividends \( g \).

20. There are several steps to carry out. First, given that the earnings retention rate is 75% and the expected rate of return on future investments is 20%, we have:

\[ g = 0.75 \times 0.20 = .15 \]

Second, we compute next period’s expected cash dividend payment as:

\[ D_1 = \$14.00 \times (1 - 0.75) = \$14.00 \times 0.25 = \$3.50 \]

Third, applying the constant-growth-rate DDM, we estimate the intrinsic value of a share of QRS stock as:

\[ P_0 = \frac{D_1}{k - g} = \frac{\$3.50}{0.16 - 0.15} = \frac{\$3.50}{0.01} = \$350.00 \]

Next, we compute the level perpetuity component of the intrinsic value of a share of the stock as:

\[ P_0 = \frac{E_1}{k} = \frac{\$14.00}{0.16} = \$87.50 \]
The NPV of future investments is the difference between the two values:

\[ \$350.00 - \$87.50 = \$262.50 \]

21. See the relevant discussion in the text.

22. The expected rate of return is computed as:

\[ E(r) = (-0.10 \times 0.40) + (0.10 \times 0.40) + (0.40 \times 0.20) = (-0.04) + 0.04 + 0.08 = 0.08 \text{ (or 8\%)} \]

The standard deviation is computed as:

\[ \sigma = \sqrt{((0.08 + 0.10)^2 + (0.08 - 0.10)^2 + (0.08 - 0.40)^2)/3} = \sqrt{(0.0324 + 0.0004 + 0.1024)/3} = 0.2123 \]

23. The probability is 95% that the actual rate of return will fall within two standard deviations of the expected rate of return. So, since \( E(r) = 0.08 \) and \( \sigma = 0.2123 \), these bounds are:

\[ (0.08 - 2 \times 0.2123, \ 0.08 + 2 \times 0.2123) = (-0.3446, 0.5046) \]

24. Recall that, when mixing a single risky asset with the riskless asset, the proportion to be allocated in the risky asset \( (w) \) is given by:

\[ w = \frac{E(r) - r_f}{E(r_s) - r_f} \]

So, if \( E(r) \) and \( r_f \) are held constant, a decrease in \( E(r_s) \) increases \( w \) and thus decreases the fraction of the portfolio places in the risk-free asset. The intuition behind this is result is that with a decrease in \( E(r_s) \), more of the risky asset must be held to reach the given \( E(r) \).

25. In the Problem #24, we saw that a decrease in \( E(r_s) \) increases \( w \). So, since the standard deviation of the portfolio is given by:

\[ \sigma = \sigma_s \times w, \]

an increase in \( w \) (brought about by the decrease in \( E(r_s) \)) leads, all else equal, to an increase in \( \sigma \).

26. Recall that the equation for the risk-reward trade-off line in the \( (\sigma, E(r)) \) plane is:

\[ E(r) = r_f + \frac{E(r_s) - r_f}{\sigma_s} \sigma. \]

So, since \( E(r_s) \) enters positively in the numerator of the slope of this equation, a decrease in \( E(r_s) \) leads to a decrease in the slope of the risk-reward trade-off line.

27. Since \( E(r) = 0.10, \ E(r_s) = 0.15, \) and \( r_f = 0.04 \), we can compute:

\[ w = \frac{E(r) - r_f}{E(r_s) - r_f} = \frac{0.10 - 0.04}{0.15 - 0.04} = 0.06/0.11 = 0.5454 \text{ (or 54.54\%)} \]

28. Given \( r_f = 0.04, \ E(r_s)0.14, \) \( \sigma_s = 0.2, \) and \( \sigma = 0.05 \), we can compute:

\[ E(r) = r_f + \frac{E(r_s) - r_f}{\sigma_s} \sigma = 0.04 + \left( \frac{0.14-0.04}{0.2} \right) \times 0.05 = 0.065 \text{ (or 6.5\%)} \]

29. In this case the “safe money market fund” is the riskless asset. So, since we’re told that \( r_f = 0.04, \ E(r_s) = 0.17, \) and \( \sigma_s = 0.4 \), the equation for the risk-reward trade-off line is:
\[ E(r) = r_f + \frac{E(r_s) - r_f}{\sigma_s} \sigma = 0.04 + \left( \frac{0.17 - 0.04}{0.4} \right) \sigma = 0.04 + 0.325 \sigma \]

30. Given \( E(r) = 0.11 \), \( r_f = .03 \), and \( E(r_s) = 0.28 \), we first compute the proportion of the portfolio in the risky asset as:

\[ w = \frac{E(r) - r_f}{E(r_s) - r_f} = \frac{0.11 - 0.03}{0.28 - 0.03} = 0.8/0.25 = 0.32. \]

Then, given \( \sigma_s = 0.4 \), we have:

\[ \sigma = 0.32 \times 0.4 = 0.128, \]

since for such a portfolio, \( \sigma = w \sigma_s \).

31. First we need to compute the \( \sigma \) for this portfolio. Proceeding as in the solution for Problem #30, since \( E(r_s) = 0.13 \) and both \( E(r) \) and \( r_f \) are the same, we first compute the proportion of the portfolio in the risky asset as:

\[ w = \frac{E(r) - r_f}{E(r_s) - r_f} = \frac{0.11 - 0.03}{0.13 - 0.03} = 0.08/0.11 = 0.8. \]

Then, given \( \sigma_s = 0.09 \), we have:

\[ \sigma = 0.8 \times 0.09 = 0.072, \]

since for such a portfolio, \( \sigma = w \sigma_s \). Comparing this portfolio with the one formed in Problem #30, both of which have an expected rate of return of 11%, we see that this portfolio is the one with the lower standard deviation. This implies that the portfolio formed in Problem #30 can not be efficient. The portfolio formed in this problem might or might not be efficient; the information given does not rule out either possibility.

32. Let \( T \) refer to the “optimal combination” of risky assets. Then we first need to compute \( E(r_T) \) and \( \sigma_T \). From the information given in Problem #30 and this problem, we have:

\[ E(r_T) = w \times E(r_1) + (1 - w) \times E(r_2) = 0.9 \times 0.28 + 0.1 \times 0.13 = 0.265 \]

and:

\[ \sigma_T^2 = w^2 \times \sigma_1^2 + (1 - w)^2 \times \sigma_2^2 = 0.92 \times 0.4^2 + 0.1^2 \times 0.09^2 = 0.129681 \Rightarrow \sigma = 0.360112 \]

Then, since the equation for the risk-reward trade-off line in this case is given by:

\[ E(r) = r_f + \frac{E(r_T) - r_f}{\sigma_T} \sigma, \]

we know that the slope of the risk-reward trade-off line is:

\[ \frac{E(r_T) - r_f}{\sigma_T} \sigma_T = \frac{0.265 - 0.03}{0.360112} = 0.65 \]

33. Letting \( T \) still refer to the “optimal combination” of risky assets, we have:

\[ E(r) = 0.4 \times r_f + 0.6 \times r_T = 0.4 \times 0.03 + 0.6 \times 0.265 = 0.171 \text{ (or 17.1%)} \]

34. The “market portfolio” holds all assets in proportion to their observed market values. From the information given, the total market value of all assets is:

\$600 \text{ billion} + \$300 \text{ billion} + \$100 \text{ billion} = \$1,000 \text{ billion} \text{ (or one trillion dollars)} \Rightarrow \]
The share of the market portfolio in AAA stock = 600/1,000 = 0.60 (or 60%)
The share of the market portfolio in BBB stock = 300/1,000 = 0.30 (or 30%)
The share of the market portfolio in the riskless asset = 100/1,000 = 0.10 (or 10%)

35. According to the CAPM, in equilibrium any investor’s relative holdings of risky assets will be the same as in the market portfolio. Thus, in this case all investors will hold AAA and BBB stock in the proportion of 2 to 1 (i.e., 600/300). With $5,000 invested in the riskless asset, $75,000 is invested in risky assets. Two-thirds of this $75,000, or $50,000, will be invested in AAA stock, and one-third of this $75,000, or $24,000, will be invested in BBB stock.

36. First we need to compute the interest rate to use as the discount rate in order to calculate the PV of the bond. The relevant interest rate to use is the interest rate on the riskless asset (rf) in this problem. According to the CAPM:

\[ E(r_M) - r_f = A\sigma^2 \Rightarrow r_f = E(r_M) - A\sigma^2 \]

So, given that \( E(r_M) = 0.7 \), \( A = 4 \), and \( \sigma^2 = 0.12 \), we have:

\[ r_f = 0.7 - (4 \times 0.12) = 0.22 \]

Second, the revenue raised by the government per bond is given by the price (i.e., PV) of the bond, which is computed as:

\[ PV = \frac{\$100,000}{1.22} = \$81967.21 \]

37. From the Security Market Line, we know that, for security j:

\[ E(r_j) = r_f = \beta_j [E(r_M) - r_f] \Rightarrow \beta_j = \frac{E(r_j) - r_f}{E(r_M) - r_f} \]

Given that \( E(r_j) = 0.12 \), \( r_f = 0.04 \), and \( E(r_M) = 0.20 \), we have:

\[ \beta_j = \frac{0.12 - 0.04}{0.20 - 0.04} = 0.5 \]

38. Using the Capital Market Line, the following holds true for an efficient portfolio:

\[ E(r) = r_f + \frac{E(r_M) - r_f}{\sigma_M} \sigma. \]

Given that \( \sigma_M = 0.4 \) (note that we’re told \( \sigma_M^2 = 0.16 \)), \( E(r) = 0.12 \), \( r_f = 0.04 \), and \( E(r_M) = 0.2 \), we then have:

\[ 0.12 = 0.04 + \frac{0.2 - 0.04}{0.4} \sigma \Rightarrow \sigma = \frac{0.12 - 0.04}{0.4} = 0.2 \]

39. The explanation is given in option (a) for this problem.

40. Since portfolio A has a higher expected rate of return and the same standard deviation as the market portfolio, it lies above the Capital Market Line, which is a violation of the CAPM.

41. See Quick-Check problem #13-4.

42. Since portfolio A has a lower standard deviation and a higher expected rate of return than the market portfolio, this situation implies that the market portfolio is not efficient.
43. For an option, recall that:

\[ \text{Time Value} = \text{Option's Price} - \text{Intrinsic Value} \]

So, if the intrinsic value is zero, the time value is equal to the price.

44. Since (i) the intrinsic value of the call option is zero, and (ii) the strike price is not equal to the price of a share of the underlying stock, the intrinsic value of the corresponding put option is positive. This implies that the time value is less than the price of the option.

45. By investing $4,000 in one-year call options at a price of $10, the investor can buy 400 options. If the options expire worthless, the investor will have $100,800 ($96,000 \times 1.05) from the investment in the riskless asset, giving a return of $800 on the $100,000, which works out to a rate of return of 0.8% (\frac{800}{100,000}).