Solutions to Problems on Final Exam

1. Given \( i = 0.05 \) and \( n = 6 \), we apply the ordinary annuity \( PV \) formula to compute this fixed-income security’s price as follows:

\[
\text{Bond Price} = \frac{1 - (1.05)^{-6}}{.05} \times 150 = \$761.35
\]

2. Given the yield curve information, the \( PV \) of the three $120 payments is given by:

\[
PV = 0.97 \times 120 + 0.93 \times 120 + 0.91 \times 120 = 120 \times (0.97 + 0.93 + 0.91) = \$337.20
\]

This \( PV \) is the price of the fixed-income security.

3. Since payments are made each period, it clearly is not a zero-coupon bond. Also, since the payment is the same for each period, it is not a coupon bond (recall that, for a coupon bond, the final payment is larger than the preceding payments, since the final payment is made up of the last coupon payment and the face value). This implies that it is neither a premium, discount, nor par bond. Thus, option (e) is the correct choice.

4. Straightforward calculations show that the yields to maturity for these zero-coupon bond prices are as follows:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>YTM</th>
<th>Compute ( i ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>4%</td>
<td>( \frac{1}{(1 + i)^1} = 0.9615 )</td>
</tr>
<tr>
<td>2 years</td>
<td>5%</td>
<td>( \frac{1}{(1 + i)^2} = 0.9070 )</td>
</tr>
<tr>
<td>3 years</td>
<td>6%</td>
<td>( \frac{1}{(1 + i)^3} = 0.8386 )</td>
</tr>
<tr>
<td>4 years</td>
<td>7%</td>
<td>( \frac{1}{(1 + i)^4} = 0.7629 )</td>
</tr>
</tbody>
</table>

So, the yield curve is upward-sloping for the range of maturities depicted.

5. We have \( i = 0.05 \), \( n = 25 \), \( FV = \$990 \), and the price of the bond is given by the \( PV \) of this \( FV \) to be received 25 years from now. Since this is a pure discount bond, the interest rate on it is the appropriate interest rate to use as the discount rate. So, the price of the bond is given by:

\[
\text{Bond Price} = \frac{\$990}{(1.05)^{25}} = \$292.35
\]

6. Proceeding as in the solution for Problem #5, we still have \( i = 0.05 \) and \( FV = \$990 \), but now \( n = 23 \). So, the price of the bond is given by:

\[
\text{Bond Price} = \frac{\$990}{(1.05)^{23}} = \$322.32
\]

7. Given the solutions to Problems #5 and #6, we need to solve for \( g \) in the following:

\[
\$322.32 = (1 + g)^2 \times \$292.35 \Rightarrow (1 + g) = \sqrt{\frac{322.32}{292.35}} \Rightarrow g = 0.05
\]
8. Recall that:

\[
\text{Current Yield} = \frac{\text{Coupon Payment}}{\text{Bond Price}}.
\]

To compute this, we need to know the Coupon Payment. Since the face value is $1100 and the coupon rate is 7%, the coupon payment is $77 ($1100 \times 0.07$). So:

\[
\text{Current Yield} = \frac{77}{950} = 0.081 \text{ (or 8.1%).}
\]

9. Given that the coupon rate is 12%, the face value is $1,100, and the current price of the bond is $950, the yield to maturity is the value of \(i\) that solves the following:

\[
950 = \frac{1,100 \times (1.12)}{1+i} \Rightarrow i = \frac{1.232}{950} - 1 = 0.2968 \text{ (or 29.68%).}
\]

10. First we need to find the price of the bond six months ago. It’s given by the following \(PV\) calculation, using \(n = 25, i = 0.03, \text{ and } FV = 1,000:\n
\[
PV = \frac{1,000}{(1.03)^{25}} = 477.61
\]

Next we compute the price of the bond today, using \(n = 24.5\) (since one-half of a year has passed), \(i = 0.04\) and keeping \(FV\) unchanged:

\[
PV = \frac{1,000}{(1.04)^{24.5}} = 382.55
\]

Then the rate of return is computed as:

\[
\text{Rate of return} = \frac{\text{coupon} + \text{change in bond price}}{\text{initial bond price}}
\]

Since this is a zero-coupon bond, the coupon payment equals 0, so we have:

\[
\text{Rate of return} = \frac{382.55 - 477.61}{477.61} = \frac{-95.06}{477.61} = -0.1990 \text{ (or -19.90%).}
\]

11. We’re given that \(D_1 = 5, k = 0.10, \text{ and } P_0 = 20\). So, according to the constant- growth-rate DDM, the expected growth rate of dividends is the value of \(g\) that solves the following:

\[
20 = \frac{5}{0.10 - g} \Rightarrow g = 0.10 - \frac{5}{20} = -0.15 \text{ (or -15.00%).}
\]

12. We’re given that \(D_0 = 4, g = 0.07, \text{ and } k = 0.11\). So, according to the constant- growth-rate DDM, the intrinsic value of the stock \(P_0\) is given by:

\[
P_0 = \frac{4 \times (1+0.07)}{0.11 - 0.07} = \frac{4.28}{0.04} = 107.00
\]

13. We still have \(D_0 = 4\) and \(g = 0.07\). But now we’re told that \(P_0 = 85.60\) and we need to solve for \(k\). The solution is given by:

\[
85.60 = \frac{4 \times (1+0.07)}{k-0.07} \Rightarrow k = \frac{4.28 + (85.60 \times 0.07)}{85.60} = 0.12 \text{ (or 12%).}
\]

14. By definition, we have:

\[
g = \text{earnings retention rate} \times \text{rate of return on new investments} = 0.75 \times 0.18 = 0.135 \text{ (or 13.5%).}
\]
15. From Problem #14 we know that $E_1 = $5 and the earnings retention rate is 75%. This implies that:

$$D_1 = 0.25 \times 5 = $1.25,$$

which, along with other information given in Problem #14, implies:

$$P_0 = \frac{\$1.25}{0.15 - 0.135} = $83.33$$

16. The stock price (whose current value was found, in Problem #14, to be $83.33) is expected to grow at the same rate as dividends (which, in Problem #13 was found to be 13.5% per year). So:

$$P_1 = P_0 \times (1 + g) = $83.33 \times 1.135 = $94.58$$

17. According to the constant-growth-rate DDM:

$$P_9 = \frac{D_{10}}{k - g} = \frac{13.25}{0.15 - 0.135} = $189.29$$

18. In Problem #17, we found that $P_9 = $189.29. We now use the market capitalization rate $k = 15\%$ to get the present value of $189.29 received 9 years from now:

$$P_0 = \frac{189.29}{(1.15)^9} = $53.81$$

19. If $P_0 = $77.00, then, given that the market capitalization rate is 15%, $P_9 = (1.15)^9 \times 77.00 = $270.88$. Next, we need to find the value of $g$ that solves the following:

$$270.88 = \frac{13.25}{0.15 - g} \Rightarrow g = 0.15 - \frac{13.25}{270.88} = 0.101085 \text{ (10.1085%) }$$

20. There are several steps to carry out. First, given that the earnings retention rate is 85% and the expected rate of return on future investments is 18%, we have:

$$g = 0.85 \times 0.18 = .153$$

Second, we compute next period’s expected cash dividend payment as:

$$D_1 = 12.00 \times (1 - 0.85) = 12.00 \times 0.15 = $1.80$$

Third, applying the constant-growth-rate DDM, we estimate the intrinsic value of a share of QRS stock as:

$$P_0 = \frac{D_1}{k - g} = \frac{1.80}{0.17 - 0.153} = $105.88$$

Next, we compute the level perpetuity component of the intrinsic value of a share of the stock as:

$$P_0 = \frac{E_1}{k} = \frac{12.00}{0.17} = $70.59$$

The NPV of future investments is the difference between the two values:

$$105.88 - 70.59 = $35.29$$

21. See the relevant discussion in the text.

22. The expected rate of return is computed as:

$$E(r) = \frac{-0.25 + 0.25 + 0.50}{3} = 0.1667 \text{ (or } 16.67\%)$$
The standard deviation is computed as:
\[
\sigma = \sqrt{((0.1667 + 0.25)^2 + ((0.1667 - 0.25)^2 + (0.1667 - 0.50)^2)/3 = 0.3118}
\]

23. This is an example of “insurance,” since the purchase of the put option represents the payment of premium to avoid a possible loss (in this case, the loss avoided is the one caused by a possible depreciation of the U.S. dollar against the Euro), and the purchase of the put option does not prevent you from a possible gain (in this case, the gain not given up is the one caused by a possible appreciation of the dollar against the Euro).

24. Recall that, when mixing a single risky asset with the riskless asset, the proportion to be allocated in the risky asset \(w\) is given by:

\[
w = \frac{E(r) - r_f}{E(r_s) - r_f}
\]

So, if \(E(r)\) and \(r_f\) are held constant, an increase in \(E(r_s)\) decreases \(w\).

25. In the Problem #24, we saw that an increase in \(E(r_s)\) decreases \(w\). So, since the standard deviation of the portfolio is given by:

\[
\sigma = \sigma_s \times w,
\]

a decrease in \(w\) (brought about by the increase in \(E(r_s)\)) leads, all else equal, to a decrease in \(\sigma\).

26. Recall that the equation for the risk-reward trade-off line in the \((\sigma, E(r))\) plane is:

\[
E(r) = r_f + \frac{E(r_s) - r_f}{\sigma_s} \sigma.
\]

So, since \(E(r_s)\) enters positively in the numerator of the slope of this equation, an increase in \(E(r_s)\) leads to an increase in the slope of the risk-reward trade-off line.

27. Since \(E(r) = 0.12, E(r_s) = 0.18,\) and \(r_f = 0.07,\) we can compute:

\[
w = \frac{E(r) - r_f}{E(r_s) - r_f} = \frac{0.12 - 0.07}{0.18 - 0.07} = 0.05/0.11 = 0.4545 \text{ (or 45.45\%)}
\]

28. Given \(r_f = 0.08, E(r_s) = 0.16, \sigma_s = 0.4,\) and \(\sigma = 0.15,\) we can compute:

\[
E(r) = r_f + \frac{E(r_s) - r_f}{\sigma_s} \sigma = 0.08 + (0.16 - 0.08)/0.4 \times 0.15 = 0.11 \text{ (or 11\%)}
\]

29. In this case the “safe money market fund” is the riskless asset. So, since we’re told that \(r_f = 0.055, E(r_s) = 0.19,\) and \(\sigma_s = 0.2,\) the equation for the risk-reward trade-off line is:

\[
E(r) = r_f + \frac{E(r_s) - r_f}{\sigma_s} \sigma = 0.055 + (0.19 - 0.055)/0.2 \sigma = 0.055 + 0.675\sigma
\]

30. Given \(E(r) = 0.1, r_f = .04,\) and \(E(r_s) = 0.24,\) we first compute the proportion of the portfolio in the risky asset as:

\[
w = \frac{E(r) - r_f}{E(r_s) - r_f} = \frac{0.1 - 0.04}{0.24 - 0.04} = 0.06/0.2 = 0.3.
\]

Then, given \(\sigma_s = 0.3,\) we have:

\[
\sigma = 0.3 \times 0.3 = 0.09,
\]
since for such a portfolio, $\sigma = w\sigma_s$.

31. First we need to compute the $\sigma$ for this portfolio. Proceeding as in the solution for Problem 30, since $E(r_s) = 0.16$ and both $E(r)$ and $r_f$ are the same, we first compute the proportion of the portfolio in the risky asset as:

$$w = \frac{E(r) - r_f}{E(r_s) - r_f} = \frac{0.1 - 0.04}{0.16 - 0.04} = \frac{0.06}{0.12} = 0.5.$$ 

Then, given $\sigma_s = 0.22$, we have:

$$\sigma = 0.5 \times 0.22 = 0.11,$$

since for such a portfolio, $\sigma = w\sigma_s$. Comparing this portfolio with the one formed in Problem 30, both of which have an expected rate of return of 10%, we see that this portfolio is the one with the higher standard deviation. This implies that the portfolio formed in this problem can not be efficient. The portfolio formed in Problem 30 might or might not be efficient; the information given does not rule out either possibility.

32. Let $T$ refer to the “optimal combination” of risky assets. Then we first need to compute $E(r_T)$ and $\sigma_T$. From the information given in Problem 30 and this problem, we have:

$$E(r_T) = w \times E(r_1) + (1 - w) \times E(r_2) = 0.75 \times 0.24 + 0.25 \times 0.16 = 0.22$$

and:

$$\sigma_T^2 = w^2 \times \sigma_1^2 + (1 - w)^2 \times \sigma_2^2 = 0.75^2 \times 0.3^2 + 0.25^2 \times 0.22^2 = 0.05365 \Rightarrow \sigma = 0.23162$$

Then, since the equation for the risk-reward trade-off line in this case is given by:

$$E(r) = r_f + \frac{E(r_T)}{\sigma_T} \sigma,$$

we know that the slope of the risk-reward trade-off line is:

$$\frac{E(r_T)}{\sigma_T} = \frac{0.22 - 0.04}{0.23162} = 0.78$$

33. Letting $T$ still refer to the “optimal combination” of risky assets, we have:

$$E(r) = 0.25 \times r_f + 0.75 \times r_T = 0.25 \times 0.04 + 0.75 \times 0.22 = 0.175 \text{ (or 17.5%) }$$

34. The “market portfolio” holds all assets in proportion to their observed market values. From the information given, the total market value of all assets is:

$$\$300 \text{ billion } + \$200 \text{ billion } + \$100 \text{ billion } = \$600 \text{ billion } \Rightarrow$$

- The share of the market portfolio in AAA stock = $300/600 = 0.50$ (or 50%)
- The share of the market portfolio in BBB stock = $200/600 = 0.3333$ (or 33.33%)
- The share of the market portfolio in the riskless asset = $100/600 = 0.1667$ (or 16.67%)

35. According to the CAPM, in equilibrium any investor’s relative holdings of risky assets will be the same as in the market portfolio. Thus, in this case all investors will hold AAA and BBB stock in the proportion of 3 to 2 (i.e., 300/200). With $5,000 invested in the riskless asset, $70,000 is invested in risky assets. 60% of this $70,000, or $42,000, will be invested in AAA stock, and 40% of this $70,000, or $28,000, will be invested in BBB stock.
36. First we need to compute the interest rate to use as the discount rate in order to calculate the $PV$ of the bond. The relevant interest rate to use is the interest rate on the riskless asset ($r_f$) in this problem. According to the CAPM:

$$E(r_M) - r_f = A\sigma^2 \Rightarrow r_f = E(r_M) - A\sigma^2$$

So, given that $E(r_M) = 0.8$, $A = 3$, and $\sigma^2 = 0.25$, we have:

$$r_f = 0.8 - (3 \times 0.25) = 0.05$$

Second, the revenue raised by the government per bond is given by the price (i.e., $PV$) of the bond, which is computed as:

$$PV = \frac{100,000}{1.05} = \$95,238.10$$

37. From the Security Market Line, we know that, for security $j$:

$$E(r_j) = r_f = \beta_j[E(r_M) - r_f] \Rightarrow \beta_j = \frac{E(r_j) - r_f}{E(r_M) - r_f}$$

Given that $E(r_j) = 0.18$, $r_f = 0.06$, and $E(r_M) = 0.15$, we have:

$$\beta_j = \frac{0.18 - 0.06}{0.15 - 0.06} = 1.33$$

38. Using the Capital Market Line, the following holds true for an efficient portfolio:

$$E(r) = r_f + \frac{E(r_M) - r_f}{\sigma_M} \sigma.$$ 

Given that $\sigma_M = 0.3$ (note that we’re told $\sigma^2_M = 0.09$), $E(r) = 0.18$, $r_f = 0.06$, and $E(r_M) = 0.15$, we then have:

$$0.18 = 0.06 + \frac{0.15 - 0.06}{0.3} \sigma \Rightarrow \sigma = \frac{0.18 - 0.06}{0.3} = 0.4$$

39. The explanation is given in option (d) for this problem.

40. Since portfolio A has a lower expected rate of return and the same standard deviation as the market portfolio, portfolio A is not efficient. This implies that it lies below the Capital Market Line.

41. The slope of the CML is clearly: $\frac{0.06}{0.072} = 0.5$. Then, given that $r_f = 0.05$ and the standard deviation of A is $0.06$, the expected rate of return on A can not be greater than 0.08. But since we’re told that $E(r_A) = 0.09$, portfolio A lies above the CML, which is a contradiction of the CAPM since this implies that the CML is not efficient.

42. Since portfolio A has a lower standard deviation and a higher expected rate of return than the market portfolio, this situation implies that the market portfolio is not efficient.

43. Check the definitions of the terms in options (a), (b), and (c). The terms listed in options (d) and (e) were never defined for use in this context.

44. With 1,500 US dollars the investor can currently buy 2,400 Canadian dollars, since the exchange rate is 1.60 Canadian dollars to the U.S. dollar ($1,500 \times 1.6 = 2,400$). If the Canadian dollar is bought, in one year the investor will have 2,544 Canadian dollars, since the Canadian government bond is 6.0% ($2,400 \times 1.06 = 2,544$). To compute the realized U.S. dollar rate of return on the Canadian government bond, we have to convert these 2,544 Canadian dollars to U.S. dollars at the assumed one-year-ahead exchange rate of 1.40 Canadian dollars to the U.S. dollar. To do this compute:
U.S. dollar value of 2,544 Canadian dollars in one year = 2,544/1.40 = 1,817.14

Then, we compute the realized U.S. dollar rate of return on the Canadian government bond as:

\[
\frac{1,817.14 - 1,500}{1,500} = 0.2114 \text{ (or 21.14%)}
\]

45. Let \( x \) be the Canadian dollar / U.S. dollar exchange rate at year’s end. Then, we need to find the value of \( x \) that solves the following equation:

\[
\frac{(2,544/x) - 1,500}{1,500} = 0.05 \Rightarrow x = \frac{2,544}{1,500} = 1.6308
\]

46. With monthly compounding, \( PV = \$900, \ i = 9\%, \) and \( n = 25: \)

\[
FV = \$900 \times (1 + (0.09/12))^{25 \times 12} = \$900 \times (1.0075)^{300} = \$8,467.57
\]

47. With quarterly compounding, \( PV = \$1,000, \ i = 18.4\%, \) and \( n = 3.5 \) years:

\[
FV = \$1,000 \times (1 + (0.184/4))^{3.5 \times 4} = \$1,000 \times (1.046)^{14} = \$1,876.91
\]

48. With weekly compounding and \( i = 17.59\%: \)

\[
EFF = (1 + (0.26/52))^{52} - 1 = (1.005)^{52} - 1 = 0.2961 \text{ (or 29.61%)}
\]

49. Since we have \( PV = \$950, \ FV = \$1,175, \) and \( n = 5, \) the internal rate of return is the value of \( i \) that solves the following:

\[
950 = \frac{1,175}{(1+i)^5} \Rightarrow i = (1.175/950)^{0.2} - 1 = 0.0434 \text{ (or 4.34%)}
\]

50. Note that:

(a) Since 1 BP = $2.50 \Rightarrow $1 = 0.4 BP

(b) Since 1 CD = $0.80 \Rightarrow $1 = 1.25 CD

Together, (a) and (b) imply 0.4 BP = 1.25 CD \Rightarrow CD = (0.4/1.25) BP = 0.32 BP