Solutions to Problems from Final Exam

1. This is a straightforward application of the principle that there is an inverse relationship between the price of a fixed income security (e.g., a bond) and market interest rates.

2. If \( i = 4\% \), the price of the bond is given by:

\[
\text{Bond Price} = \frac{1 - (1.04)^{-5}}{.04} \times 120 = $534.22
\]

If \( i = 7\% \), the price of the bond is given by:

\[
\text{Bond Price} = \frac{1 - (1.07)^{-5}}{.07} \times 120 = $492.02
\]

So, as \( i \) goes from 4\% to 7\%, the price of the bond changes from $534.22 to $492.02, creating a drop in market value equal to:

\[
$534.22 - $492.02 = $42.20
\]

3. As an implication of LOOP, the coupon bond’s one-year return must be 7\%. Given that that the holder of the coupon bond will receive a cash flow equal to $1,188 (since the face value equals $1,110 and the coupon rate is 8\%), the price of the bond is the value of \( P \) that solves the following:

\[
\frac{1,188}{P} = 1.07 \Rightarrow P = \frac{1,188}{1.07} = $1,110.28
\]

4. Since the bond’s price is currently greater than its face value, it is, by definition, a premium bond.

5. For discount bonds:

\[
\text{Yield to Maturity} > \text{Current Yield} > \text{Coupon Rate}
\]

6. The solution is the value of \( i \) that solves the following:

\[
(1 + i)^4 = \frac{1}{.58} \Rightarrow i = (\frac{1}{.58})^{0.25} - 1 \Rightarrow i = 3.25\%
\]

7. We use the zero-coupon bond prices to compute the present value of each year’s cash flow:

- \( PV \) of first year’s cash flow = \( 5 \times 0.98 = 4.90 \)
- \( PV \) of second year’s cash flow = \( 5 \times 0.94 = 4.70 \)
- \( PV \) of third year’s cash flow = \( 105 \times 0.92 = 96.60 \)

So, since the total present value equals 4.90 + 4.70 + 96.60 = 106.20, the price of the 3-year coupon bond is $106.20.
8. Using the zero-coupon bond prices it’s straightforward to compute the yield to maturity on a zero-coupon at each maturity. The results are:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Yield to Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>2.041%</td>
</tr>
<tr>
<td>2 years</td>
<td>3.142%</td>
</tr>
<tr>
<td>3 years</td>
<td>2.818%</td>
</tr>
<tr>
<td>4 years</td>
<td>3.250%</td>
</tr>
</tbody>
</table>

Since the YTM on the 2-year bond is higher than the YTM on both the 1-year and the 3-year bond, the yield curve is neither flat, nor upward-sloping, nor downward-sloping throughout.

9. Given that the coupon rate is 15%, the face value is $950, and the current price of the bond is $1,020, the yield to maturity is the value of $i$ that solves the following:

$$1,020 = \frac{950 \times (1.15)}{1+i} = \frac{1,092.5}{1+i} \Rightarrow i = \frac{1,092.5}{1,020} - 1 = 7.11\%$$

10. We’re given that $D_1 = $6, $k = 11\%$, and $P_0 = $22. So, according to the constant-growth-rate DDM, the expected growth rate of dividends is the value of $g$ that solves the following:

$$22 = \frac{6}{.11-g} \Rightarrow g = .11 - \frac{6}{22} = -0.1627$$ (or -16.27%)

11. We’re given that $D_0 = $3, $g = 6\%$, and $k = 12\%$. So, according to the constant-growth-rate DDM, the intrinsic value of the stock ($P_0$) is given by:

$$P_0 = \frac{3 \times (1+.06)}{.12-.06} = \frac{3.18}{.06} = $53.00$$

12. We still have $D_0 = $3, $g = 6\%$. But now we’re told that $P_0 = $35.33 and we need to solve for $k$. The solution is given by:

$$35.33 = \frac{3 \times (1+.06)}{k-.06} \Rightarrow k = \frac{3.18 + (35.33 \times .06)}{35.33} = 15\%$$

13. By definition, we have:

$$g = \text{earnings retention rate} \times \text{rate of return on new investments} = .85 \times .16 = .136 = 13.6\%$$

14. From Problem #13 we know that $E_1 = $4 and the earnings retention rate is 85%. This implies that:

$$D_1 = .15 \times 4 = .6$$, which, along with other information given in Problem #13, implies:

$$P_0 = \frac{6}{.14-.136} = $150$$

15. The stock price (whose current value was found, in Problem #14, to be $150) is expected to grow at the same rate as dividends (which, in Problem #13 was found to be 13.6\% per year). So:

$$P_1 = P_0 \times (1+g) = 150 \times 1.136 = $170.40$$

16. According to the constant-growth-rate DDM:

$$P_4 = \frac{D_5}{k-g} = \frac{1.50}{.13-.09} = $37.50.$$
17. In Problem #16, we found that $P_4 = 37.50. We now use the market capitalization rate $k = 13\%$ to get the present value of $37.50$ received 4 years from now:

$$P_0 = \frac{37.50}{(1.13)^4} = 23.00$$

18. If $P_0 = 110.40$, then, given that the market capitalization rate is 13%, $P_4 = (1.13)^4 \times 110.40 = 180.00$. Next, we need to find the value of $g$ that solves the following:

$$180.00 = \frac{1.50}{1.13 - g} \Rightarrow g = 1.13 - \frac{1.5}{180.00} = .12167 (12.167\%)$$

19. There are several steps to carry out. First, given that the earnings retention rate is 80% and the expected rate of return on future investments is 15%, we have:

$$g = .80 \times .15 = .12$$

Second, we compute next period’s expected cash dividend payment as:

$$D_1 = 11 \times (1 - .80) = 11 \times .20 = 2.2$$

Third, applying the constant-growth-rate DDM, we estimate the intrinsic value of a share of QRS stock as:

$$P_0 = \frac{D_1}{k - g} = \frac{2.2}{.16 - .12} = \frac{2.2}{.04} = 55.00$$

Next, we compute the level perpetuity component of the intrinsic value of a share of the stock as:

$$P_0 = \frac{E_1}{k} = \frac{11}{.16} = 68.75$$

The NPV of future investments is the difference between the two values:

$$55.00 - 68.75 = -13.75$$

20. This is a clear example of hedging.

21. Using the definition of expected rate of return:

$$E(r) = \sum_{i=1}^{3} P_i r_i = (.15 \times .04) + (.12 \times .06) + (.08 \times .9) = 8.52\%$$

22. Recall that the equation for the risk-reward trade-off line is given by:

$$E(r) = r_f + \frac{E(r_s) - r_f}{\sigma_s} \sigma$$

where

$E(r_s)$ is the expected rate of return on the risky asset and $\sigma_s$ is the standard deviation of the distribution of the rate of return on the risky asset. So, as $r_f$ increases, both: (a) the intercept increases; and (b) the slope decreases.

23. From the equation for $E(r)$ for the portfolio, we know that:

$$w = \frac{E(r) - r_f}{E(r_s) - r_f} = \frac{.10 -.04}{.13 -.04} = .06 \div .09 = 66.67\%$$
24. Recall that the equation for the risk-reward trade-off line is given by:

\[ E(r) = r_f + \frac{E(r_s) - r_f}{\sigma_s} \sigma. \]

So, as \( \sigma_s \) increases, the slope clearly decreases.

25. First, let's calculate the proportion of the portfolio allocated to Risky Asset 1. From the equation for \( E(r) \) for the portfolio, we know that:

\[ w = \frac{E(r) - r_f}{E(r_s)} = \frac{.11 -.05}{.13 -.05} = \frac{.06}{.08} = 75.00\% \]

Second, for portfolios formed by mixing a single risky asset with the riskless asset, we know that:

\[ \sigma = \sigma_s \times w = 0.20 \times 0.75 = 0.15 \]

26. Following the solution for Problem #25, let's first calculate the proportion of the portfolio allocated to Risky Asset 2. From the equation for \( E(r) \) for the portfolio, we know that:

\[ w = \frac{E(r) - r_f}{E(r_s)} = \frac{.11 -.05}{.11 -.05} = \frac{.06}{.06} = 100.00\% \]

This result should come as no surprise, since the expected rate of return on Risky Asset 2 is equal to the expected rate of return on the portfolio. Without further calculation we clearly know that the standard deviation of the portfolio is equal to the standard deviation of Risky Asset 2, i.e., 0.17. But to see that this result is confirmed by the standard calculations, note that:

\[ \sigma = \sigma_s \times w = 0.17 \times 1.00 = 0.17 \]

27. The two portfolios have the same expected return. By definition, the one with the higher standard deviation (i.e., the one formed in Problem #26) can't be an efficient portfolio. The portfolio formed in Problem #25 might or might not be efficient; none of the information given rules out either possibility.

28. Given the information we have about the probability distributions of these two risky assets:

\[ E(r) = w \times E(r_1) + (1 - w) \times E(r_2) = .55 \times .13 + .45 \times .11 = 12.1\% \]

29. Given the information we have about the probability distributions of these two risky assets:

\[ \sigma^2 = w^2 \times \sigma_1^2 + (1 - w)^2 \times \sigma_2^2 = .55^2 \times .20^2 + .45^2 \times .17^2 = 0.01795 \Rightarrow \sigma = .134 \]

30. Let \( T \) refer to the “optimal combination” of risky assets. Then, from Problems #28 and #29, we know that:

\[ E(r_T) = 12.1\% \text{ and } \sigma_T = .134. \]

Since the equation for the risk-reward trade-off line in this case is given by:

\[ E(r) = r_f + \frac{E(r_T) - r_f}{\sigma_T} \sigma, \]

we know that the slope of the risk-reward trade-off line is:

\[ \frac{E(r_T) - r_f}{\sigma_T} = \frac{.121 -.05}{.134} = 0.53 \]
31. Letting $T$ still refer to the “optimal combination” of risky assets, we have:

$$E(r) = 0.3 \times r_f + 0.7 \times r_T = 0.3 \times 0.05 + 0.7 \times 0.121 = 0.015 + 0.0847 = 9.97\%$$

32. The “market portfolio” holds all assets in proportion to their observed market values. From the information given, the total market value of all assets is:

$40 \text{ billion} + 8 \text{ billion} + 2 \text{ billion} = 50 \text{ billion} \Rightarrow$

The share of the market portfolio in AAA stock = $40/50 = 80\%$

The share of the market portfolio in BBB stock = $8/50 = 16\%$

The share of the market portfolio in the riskless asset = $2/50 = 4\%$

33. According to the CAPM, in equilibrium any investor’s relative holdings of risky assets will be the same as in the market portfolio. Thus, in this case all investors will hold AAA and BBB stock in the proportion of 4 to 1 (i.e., 40/10). With $10,000 invested in the riskless asset, $150,000 is invested in risky assets. 80% of this $150,000, or $120,000, will be invested in AAA stock, and 20% of this $150,000, or $30,000, will be invested in BBB stock.

34. In the CAPM, the equilibrium risk premium on the market portfolio is equal to the variance of the market portfolio times a weighted average of the degree of risk aversion of the holders of wealth($A$):

$$E(r_M) - r_f = A\sigma_M^2 = 1.5 \times 0.3^2 = 1.5 \times 0.009 = 0.135$$

35. With the information given, the equilibrium risk premium on the market portfolio is given by:

$$E(r_M) - r_f = A\sigma_M^2 = 2.0 \times 0.5^2 = 2.0 \times 0.25 = 0.5$$

Next, the slope of the CML is given by:

$$\frac{E(r_M) - r_f}{\sigma_M} = \frac{0.5}{0.3} = 1.0$$

36. Recall that:

$$E(r_j) - r_f = \beta_j[E(r_M) - r_f]$$

So, given that $r_f = 5\%$, $\beta_j = 0.6$, and $E(r_j) - r_f = 9\%$, we have:

$$E(r_j) = (0.6 \times 0.09) + 0.05 = 0.54 + 0.05 = 10.4\%$$

37. We first need to compute the variance of the market portfolio. From the information given we have:

$$\sigma_M^2 = (0.75^2 \times 0.2^2) + (.25^2 \times 0.2^2) = (0.5625 \times 0.04) + (0.0625 \times 0.04) = 0.025$$

Then, since $E(r_M) - r_f = A\sigma_M^2$, we have:

$$E(r_M) - r_f = 2.5 \times 0.025 = 0.0625 \text{ (or 6.25\%)}$$

38. The slope of the CML is clearly: $\frac{0.12}{0.24} = 0.5$. Then, given that $r_f = 0.10$ and the standard deviation of $A$ is 0.12, the expected rate of return on $A$ can not be greater than 0.16. But since we’re told that $E(r_A) = 0.18$, portfolio A lies above the CML, which is a contradiction of the CAPM since this implies that the CML is not efficient.
39. Explanation is given in option (a) for this problem.

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41. Straightforward problem.

42. With 1,000 US dollars the investor can currently buy 1,400 Canadian dollars, since the exchange rate is 1.40 Canadian dollars to the U.S. dollar. If the Canadian dollar is bought, in one year the investor will have 1,505 Canadian dollars, since the Canadian government bond is 7.5% \((1,400 \times 1.075 = 1,505)\). To compute the realized U.S. dollar rate of return on the Canadian government bond, we have to convert these 1,505 Canadian dollars to U.S. dollars at the assumed one-year-ahead exchange rate of 1.50 Canadian dollars to the U.S. dollar. To do this compute:

\[
\text{U.S. dollar value of 1,505 Canadian dollars in one year} = \frac{1,505}{1.50} = $940.63
\]

Then, we compute the realized U.S. dollar rate of return on the Canadian government bond as:

\[
\frac{\text{940.63} - 1000}{1000} = -5.94\%
\]

43. Let \(x\) be the Canadian dollar / U.S. dollar exchange rate at year’s end. Then, we need to find the value of \(x\) that solves the following equation:

\[
\frac{(1.505/x) - 1.000}{1.000} = 0.03793 \quad \Rightarrow \quad x = \frac{1.505}{1.03793} = 1.45
\]

44. With quarterly compounding, \(PV = $1,000\), \(i = 12\%\), and \(n = 30\):

\[
FV = 1,000 \times (1 + (0.12/4))^{30 \times 4} = 1,000 \times (1.03)^{120} = $34,710.99
\]

45. With weekly compounding, \(PV = 950\), \(i = 20.8\%\), and \(n = 2.5\) years:

\[
FV = 950 \times (1 + (0.208/52))^{2.5 \times 52} = 1,000 \times (1.004)^{130} = $1,596.27
\]

46. With daily compounding and \(i = 18.25\%\):

\[
EFF = (1 + (0.1825/365))^{365} - 1 = (1.0005)^{365} - 1 = 20.02\%
\]

47. Since we have \(PV = 975\), \(FV = 1,125\), and \(n = 4\), the internal rate of return is the value of \(i\) that solves the following:

\[
975 = \frac{1.125}{(1+i)^4} \quad \Rightarrow \quad i = (1.125/975)^{25} - 1 = 3.64\%
\]

48. The bank offers a higher rate of return.

49. Note that:

(a) Since 1 BP = $3.20 \(\Rightarrow\) $1 = 0.3125 BP

(b) Since 1 FF = $0.20 \(\Rightarrow\) $1 = 5 FF

Together, (a) and (b) imply 0.3125 BP = 5 FF \(\Rightarrow\) FF = \((0.3125/5)\) BP = 0.0625 BP

50. Since 575 FF = $50 \(\Rightarrow\) FF = $50/575 = $0.09