**Weighted and Unweighted Means ANOVA©**

Please read my document Weighted Means and Unweighted Means One-Way ANOVA before continuing on with this document. As explained there, the distinction between the weighted means ANOVA and the unweighted means ANOVA becomes much more important in factorial ANOVA than it is in one-way ANOVA.

**Weighted Means ANOVA with Unequal, Proportional Cell *n*’s**

**Data Set “Int”** (from Howell, 3rd ed., page 412)[[1]](#footnote-1)

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Male | | |  | Female | | |  | Marginal Means | |  |
|  | ΣX | *M* | *n* |  | ΣX | *M* | *n* |  | Weighted | Unweighted | *n* |
| School 1 | 1550 | 155 | 10 |  | 2200 | 110 | 20 |  | 125 | 132.5 | 30 |
| School 2 | 2700 | 135 | 20 |  | 4800 | 120 | 40 |  | 125 | 127.5 | 60 |
| Marginal |  |  |  |  |  |  |  |  |  |  |  |
| Weighted |  | 141.6 | 30 |  |  | 116.6 | 60 |  |  |  | 90 |
| Unweighted |  | 145 |  |  |  | 115 |  |  |  |  |  |

Note that there is an **interaction** here. The simple main effect of gender at School 1 = (155 - 110) = 45 does not equal that at School 2 = (135 - 120) = 15.

Note that the **cell *n*’s are proportional**. For each cell *χ2*= 0, O = E. See the table, below, of the expected cell counts were the rows independent of the columns. Note that in every cell the expected frequency is exactly equal to the observed frequency.

|  |  |  |
| --- | --- | --- |
|  | Sex | |
| School | Male | Female |
| 1 | 10 = 30(30) / 90 | 20 = 60(30) / 90 |
| 2 | 20 = 30(60) / 90 | 40 = 60(60) / 90 |

Look at the **main effect of school**. Using **weighted (by sample size) means**,

*M1* = [10(155) + 20(110)] / 30 = 125 = *M2* = [2700 + 4800] / 60. Since the two marginal means are exactly equal, there is absolutely no main effect of school. For gender, there is a main effect of (141.6 - 116.6) = 25.

What if we decide to weight all cell means equally? For example, we decide that we wish to weight the male means the same as the female means and School 1 means the same as School 2’s. This would be quite reasonable if our obtaining more female data than male and more School 2 data than School 1 was due to “chance” and we wished to generalize our findings to a population with 50% male students, 50% female students and 50% enrollment in School 1, 50% in School 2. We compute **“unweighted” (equally weighted) marginal means** as means of means. For the main effect of school (155 + 110) / 2 = 132.5, (135 + 120) / 2 = 127.5, and the main effect is (132.5 - 127.5) = 5. This is not what we found with a weighted means approach, which indicated absolutely no effect of school. Note that the size of the main effect of gender also varies with method of weighting the means.

What if there were **no interaction**? For example,

**Data Set ∅**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Male | | |  | Female | | |  | Marginal Means | |  |
|  | ΣX | *M* | *n* |  | ΣX | *M* | *n* |  | Weighted | Unweighted | *n* |
| School 1 | 1550 | 155 | 10 |  | 2800 | 140 | 20 |  | 145 | 147.5 | 30 |
| School 2 | 2700 | 135 | 20 |  | 4800 | 120 | 40 |  | 125 | 127.5 | 60 |
| Marginal |  |  |  |  |  |  |  |  |  |  |  |
| Weighted |  | 141.6 | 30 |  |  | 126.6 | 60 |  |  |  | 90 |
| Unweighted |  | 145 |  |  |  | 130 |  |  |  |  |  |

(155 - 140) = (135 - 120) → no interaction. The main effect for school is (145 - 125) = 20 with weighted means, = (147.5 - 127.5) = 20 for unweighted means. Choice of weighting method also has no effect on the main effect of gender.

We have seen that even with proportional cell *n*’s the row and column effects are not independent of any interaction effects present. If an interaction is present with such data, choice of weighting techniques affects the results.

**Computation of Weighted Means ANOVA Using Data Set “Int”**

*SSTOT* = 81000 (given)











|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | *SS* | *df* | *MS* | *F* | *p* |
| School | 0 | 1 | 0 | 0.0 | 1.000 |
| Gender | 12500 | 1 | 12500 | 16.6 | < .001 |
| Interaction | 4000 | 1 | 4000 | 5.3 | .024 |
| Error | 64500 | 86 | 750 |  |  |
| Total | 81000 | 89 |  |  |  |

**Interaction Analysis:**



*F*(1, 86) = 13500 / 750 = 18, *p* < .001.



*F*(1, 86) = 3000 / 750 = 4, *p* = .049.

Significant gender effects at both schools, but a greater difference between male students and female students at School 1 than at School 2.

------------------------------------ OR -------------------------------------



*F*(1, 86) = 2666.6 / 750 = 3.5, *p* = .06.



*F*(1, 86) = 1333.3 / 750 = 1.7, *p* = .19.

Nonsignificant school differences for each gender, but trends in opposite directions [Sch 1 > Sch 2 for male students, Sch 1 < Sch 2 for female students].

**Traditional Unweighted Means ANOVA**

One simple way to weight the cell means equally involves using the harmonic mean. In this case we compute: 

For the data set “Int” (School x Gender), retain the previous sums and *n*’s.



We now adjust cell totals by multiplying cell means ( *M* ) by harmonic sample size, .

|  |  |  |  |
| --- | --- | --- | --- |
|  | Male ΣY | Female ΣY | Marginal Total |
| School 1 | 2755.5 | 1955.5 | 4711.1 |
| School 2 | 2400 | 2133.3 | 4533.3 |
| Marginal Total | 5155.5 | 4088.8 | 9244.4 |











To find the *SSE*, find for each cell  and then sum these across cells.

Assume the below cell sums and *n’*s.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | School 1 | |  | School 2 | |
|  | Male | Female |  | Male | Female |
| ΣX | 1,550 | 2,200 |  | 2,700 | 4,800 |
| ΣX2 | 248,000 | 256,000 |  | 379,000 | 604,250 |
| *n* | 10 | 20 |  | 20 | 40 |

. .

. .

The sum = *SSE* = 64500. The *MSE* = the weighted average of the cell variances.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Source | *SS* | *df* | *MS* | *F* | *p* | |
| School | 444.4 | 1 | 444.4 | 0.59 | | .44 |
| Gender | 16,000 | 1 | 16,000 | 21.30 | | < .001 |
| Interaction | 4,000 | 1 | 4,000 | 5.30 | | .024 |
| Error | 64,500 | 86 | 750 |  | |  |

Gender Interaction Analysis









18,000 + 2,000 = 20,000 = 16,000 + 4,000

*F1* = 18000 / 750 = 24, *p* < .001. *F2* = 2000 / 750 = 2.6, *p* = .11.

There is a significant gender difference at School 1, but not at School 2.

----------------- Or, School Interaction Analysis ----------------------







3,555.5 + 888.8 = 4444.4 = 444.4 + 4,000

*Fmen* = 3555.5 / 750 = 4.74, *p* =.032. *Fwomen* = 888.8 / 750 = 1.185, *p* =.28.

There is a significant school difference for men but not for women.

**Reversal Paradox**

We have seen that the School x Gender interaction present in the body weight data (from page 412 of the 3rd edition of Howell) results in there being no main effect of school if we use unweighted means, but a (small) main effect being indicated if we use weighted means. When we modified one cell mean to remove the interaction, choice of weighting method no longer affected the magnitude of the main effects. The cell frequencies in Howell’s data were proportional, making school and gender orthogonal (independent).

Let me show you a strange thing that can happen when the cell frequencies are not proportional.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Gender | | | | |  |  |  |
|  | Male | |  | Female | |  | Marginal Means | |
| School | *M* | *n* |  | *M* | *n* |  | weighted | unweighted |
| 1 | 150 | 60 |  | 110 | 40 |  | 134 | 130 |
| 2 | 160 | 10 |  | 120 | 90 |  | 124 | 140 |

Note that there is no interaction, but that the cell frequencies indicate that gender is correlated with school (School 1 has a higher proportion of male students than does School 2). Weighted means indicate that body weight at School 1 exceeds that at School 2, but unweighted means indicate that body weight at School 2 exceeds that at School 1. Both make sense. School 1 has a higher mean body weight than School 2 because School 1 has a higher proportion of male students than does School 2, and men weigh more than women. But the men at School 2 weigh more than do the men at School 1 and the women at School 2 weigh more than do the women at School 1.

A **reversal paradox** is when 2 variables are positively related in aggregated data, but, within each level of a third variable, they are negatively related (or negatively in the aggregate and positively within each level of the third variable). Please read Messick and van de Geer’s article on the reversal paradox (*Psychol. Bull. 90*: 582-593). We have a reversal paradox here - in the aggregated data (weighted marginal means), students at School 1 weigh more than do those at School 2, but within each Gender, students at School 2 weigh more than those at School 1.

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1. These data were not included in the most recent edition of Howell. The dependent variable is body weight of the students. [↑](#footnote-ref-1)