

Multiple Regression, Data from Page 496 of Howell (6th ed.)
Analysis of Raw Data

The REG Procedure
Model: MODEL1
Dependent Variable: Overall

Number of Observations Read 50

Number of Observations Used 50

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	13.93426	2.78685	27.18	<.0001
Error	44	4.51074	0.10252		
Corrected Total	49	18.44500			

Root MSE	0.32018	R-Square	0.7554
Dependent Mean	3.55000	Adj R-Sq	0.7277
Coeff Var	9.01923		

Parameter Estimates									
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate	Squared Semi-partial Corr Type II	Squared Partial Corr Type II	Tolerance
Intercept	1	-1.19483	0.63116	-1.89	0.0649	0	.	.	.
Teach	1	0.76324	0.13292	5.74	<.0001	0.66197	0.18325	0.42836	0.41819
Exam	1	0.13198	0.16280	0.81	0.4219	0.10608	0.00365	0.01472	0.32457
Knowledge	1	0.48898	0.13654	3.58	0.0008	0.32506	0.07129	0.22570	0.67463
Grade	1	-0.18431	0.16550	-1.11	0.2715	-0.10547	0.00689	0.02742	0.61969
Enroll	1	0.00052549	0.00039008	1.35	0.1848	0.12424	0.01009	0.03961	0.65345

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Z_Overall = (Overall - 3.55) / .6135378;
Z_Teach = (Teach - 3.664) / .5321347;
Z_Exam = (Exam - 3.808) / .4931531;
Z_Knowledge = (Knowledge - 4.176) / .4078615;
Z_Grade = (Grade - 3.486) / .3510974;
Z_Enroll = (Enroll - 88) / 145.059453;
Proc Reg; Model Z_Overall = Z_Teach -- Z_Enroll;

```

Analysis of Standardized Data

The REG Procedure
Model: MODEL1
Dependent Variable: Z_Overall

Number of Observations Read 50
Number of Observations Used 50

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	37.01700	7.40340	27.18	<.0001
Error	44	11.98299	0.27234		
Corrected Total	49	48.99999			

Root MSE 0.52186 **R-Square** 0.7554
Dependent Mean 2.22322E-16 **Adj R-Sq** 0.7277
Coeff Var 2.347327E17

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	3.79846E-16	0.07380	0.00	1.0000
Z_Teach	1	0.66197	0.11528	5.74	<.0001
Z_Exam	1	0.10608	0.13086	0.81	0.4219
Z_Knowledge	1	0.32506	0.09077	3.58	0.0008
Z_Grade	1	-0.10547	0.09470	-1.11	0.2715
Z_Enroll	1	0.12424	0.09223	1.35	0.1848

Since we have standardized the data, the b weights reported here are the β weights we saw earlier.

Proc Reg; **Model** Teach = Exam -- Enroll;
Output out = Resids1 **r = Teach_Resid;**

For each subject we now have $\text{Teach Resid} = T - \hat{T}$. We have removed from Teach its overlap with all of the other predictors.

Create Residuals for Teach Predicted From All Remaining Predictors

The REG Procedure
 Model: MODEL1
 Dependent Variable: Teach

Number of Observations Read 50
Number of Observations Used 50

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	8.07275	2.01819	15.65	<.0001
Error	45	5.80245	0.12894		
Corrected Total	49	13.87520			

Root MSE 0.35909 **R-Square** 0.5818
Dependent Mean 3.66400 **Adj R-Sq** 0.5446
Coeff Var 9.80040

The R^2 for predicting Teach from the remaining predictors is .5818. Subtract that from 1 to get the tolerance reported earlier. $1 - .5818 = .4182$. Low tolerance indicates a problem with multicollinearity. There is no such problem here.

Proc Reg; **Model** Overall = Exam -- Enroll;
Output out = Resids2 **r = Overall_Resid;**

For each subject we now have $\text{Overall Resid} = O - \hat{O}$. We have removed from Overall its overlap with all of the other predictors except Teach.

Create Residuals for Overall Predicted From All Except Teach

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	10.55416	2.63854	15.05	<.0001
Error	45	7.89084	0.17535		
Corrected Total	49	18.44500			

Root MSE 0.41875 **R-Square** 0.5722

Dependent Mean 3.55000 Adj R-Sq 0.5342
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For the full model, R^2 was .7544. When we removed Teach from the model, the R^2 dropped to .5722. The value of that drop, $.7544 - .5722 = .1832$, is the squared semipartial correlation coefficient for Teach.

Proc Reg; Model Overall_Resid = Teach_resid / stb;

Use the Part of Teach Not Related to the Other Predictors To Predict the Part of Overall Not Related to those Other Predictors

The REG Procedure
Model: MODEL1
Dependent Variable: Overall_Resid Residual

Number of Observations Read 50

Number of Observations Used 50

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	3.38010	3.38010	35.97	<.0001
Error	48	4.51074	0.09397		
Corrected Total	49	7.89084			

Root MSE	0.30655	R-Square	0.4284
Dependent Mean	-1.0569E-15	Adj R-Sq	0.4164
Coeff Var	-2.90039E16		

Of the variance in Overall that is not explained by the other predictors, 42.84% (R^2) is explained by the part of Teach that is not related to the other predictors. This is the squared partial correlation coefficient for Teach that we saw earlier.

Parameter Estimates							
Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate
Intercept	Intercept	1	-9.6881E-16	0.04335	-0.00	1.0000	0
Teach_Resid	Residual	1	0.76324 ^A	0.12726	6.00	<.0001	0.65449 ^B

^AThe unstandardized slope for predicting that part of Overall that is not related to the other predictors from that part of Teach that is not related to the other predictors is .76324. We saw this earlier as the full model b weight for Teach.

^BThe standardized slope for predicting that part of Overall that is not related to the other predictors from that part of Teach that is not related to the other predictors is .65449. This is the full model partial correlation coefficient for Teach. If we square .65449, we get the squared partial correlation coefficient for Teach, .42836.

Descriptive Statistics on Overall, Teach, and Their Residuals

The MEANS Procedure

Variable	Label	Mean	Std Dev
Overall		3.5500000	0.6135378
Overall_Resid	Residual	-1.05693E-15	0.4012950
Teach		3.6640000	0.5321347
Teach_Resid	Residual	-1.15463E-16	0.3441182

$$\beta_{Teach} = b_{Teach} \left(\frac{s_{Teach}}{s_{Overall}} \right) = .76324 \left(\frac{.5321347}{.6315378} \right) = .66197$$

$$pr_{Teach} = b_{Teach} \left(\frac{s_{Teach_Resid}}{s_{Overall_Resid}} \right) = .76324 \left(\frac{.344182}{.401295} \right) = .65449$$

Proc Corr nosimple; Var Overall; With Teach_resid;

Correlation Between All of Overall and the Part of Teach Not Related
To the Other Predictors

Pearson Correlation Coefficients, N = 50 Prob > |r| under H0: Rho=0

	Overall
Teach_Resid	0.42808
Residual	0.0019

This is the sr_{Teach} , the correlation between all of Overall and that part of Teach that is not related to the other predictors. When we square it we get [.18325, the squared semipartial correlation coefficient for Teach](#), the proportion of all the variance in Overall that is uniquely explained by Teach.

[Karl L. Wuensch](#), January, 2020.