A factorial MANOVA may be used to determine whether or not two or more categorical grouping variables (and their interactions) significantly affect optimally weighted linear combinations of two or more normally distributed outcome variables. We have already studied one-way MANOVA, and we previously expanded one-way ANOVA to factorial ANOVA, so we should be well prepared to expand one-way MANOVA to factorial MANOVA. The normality and homogeneity of variance assumptions we made for the factorial ANOVA apply for the factorial MANOVA also, as does the “homogeneity of dispersion matrices” assumption (variance/covariance matrices do not differ across cells) we made in one-way MANOVA.

The output from MANOVA2.sas on my SAS Programs page can be found here. The data are from the same thesis that provided us the data for our one-way MANOVA, but this time there are two dependent variables: YEARS, length of sentence given the defendant by the mock-juror subject, and SEVERITY, a rating of how serious the subject thought the defendant’s crime was. The PA independent variable was a physical attractiveness manipulation: the female defendant presented to the mock jurors was “attractive,” “average,” or “not attractive.” The second independent variable was CRIME, the type of crime the defendant committed, a burglary (theft of items from victim's room) or a swindle (conned the male victim).

Multivariate Interactions

As in univariate factorial ANOVA, we shall generally inspect effects from higher order down to main effects. For our 3 x 2 design, the PA X CRIME effect is the highest order effect. We had some reason to expect this effect to be significant—others have found that beautiful defendants get lighter sentences than do ugly defendants unless the crime was one in which the defendant’s beauty played a role (such as enabling her to more easily con our male victim). We have had little luck replicating this interaction, and it has no significant multivariate effect here (but Roy’s greatest root, which tests only the first root, is nearly significant, $p = .07$). We shall, however, for instructional purposes, assume that the interaction is significant. What do we do following a significant multivariate interaction?

The unsophisticated way to investigate a multivariate interaction is first to determine which of the dependent variables have significant univariate effects from that interaction. Suppose that YEARS does and SERIOUS does not. We could then do univariate simple main effects analysis (or, if we were dealing with a triple or even higher order interaction, simple interaction analysis) on that dependent variable. This unsophisticated approach totally ignores the correlations among the dependent variables and the optimally weighted linear combinations of them that MANOVA worked so hard to obtain.

At first thought, it might seem reasonable to follow a significant multivariate PA X CRIME interaction with “multivariate simple main effects analysis,” that is, do two one-way MANOVAs: multivariate effect of PA upon YEARS and SERIOUS using data from the burglary level of CRIME only, and another using data from the swindle level of CRIME only (or alternatively, three one-way MANOVAs, one at each level of PA, with IV=CRIME). There is, however a serious complication with this strategy.
Were we to do two one-way MANOVAs, one at each level of Crime, to find the multivariate effect of PA upon our variables, those two MANOVAs would each construct two roots (weights for the variables) that maximize the eigenvalues for their own data. Neither set of weights would likely be the same as that that maximized the multivariate interaction, and the weights that maximize separation of the PA groups for burglary defendants are likely different from those that do the same for the swindle defendants. Now these differences in weights might well be quite interesting, but things have certainly gotten confusingly complex. We have three MANOVAs (one factorial with significant interaction, two one-way with multivariate main effects of PA holding Crime constant), and each has roots (weighted combinations of the DVs) different from the other two MANOVA’s roots. What a mess!

While I might actually get myself into the mess just described (sometimes out of such messes there emerges an interesting new way to view the data), here is an approach that I find easier to understand (perhaps because I created it—whether others have also, I know not). A significant multivariate interaction means that the effect of one grouping variable depends upon level of another grouping variable, where the multivariate outcome variable is one or more canonical variates. **Compute for each subject canonical variate scores for each root that is significant** (from the Dimension Reduction Analysis) and then do simple effects analysis on the corresponding canonical variate.

The MANOVA produced the test for a multivariate interaction by obtaining eigenvalue(s) (ratio of $SS_{effect}$ / $SS_{error}$) for the weighted linear combination(s) of the dependent variables that maximized that effect. That is, the eigenvalues for our multivariate interaction represent the ($SS_{interaction}$ / $SS_{error}$) we would obtain were we to do factorial ANOVAs on the optimally weighted linear combinations of the dependent variables produced by MANOVA. The number of such combinations (roots) created will be equal to the lesser of: a.) the univariate df for the effect and b.) the number of dependent variables. MANOVA chose those weights to maximize the multivariate interaction effect.

In the data step I computed Z scores and then used them with the total sample standardized canonical coefficients for the first root of the interaction effect to compute scores on the first root of the interaction (CV-INT1). Look at the ANOVA on this canonical variate (page 10). The $SS$ for PA X CRIME is 5.411, the error (residual) $SS$ is 108, and 5.411/108 = 0.0501 = the eigenvalue for root 1 for the multivariate interaction. Note that the interaction term for the ANOVA on this canonical variate is almost significant, $F(2, 108) = 2.71, .0714$. Also note that this test is absolutely equivalent ($F$, df, & p) to that of Roy’s Greatest Root.

Let us pretend that our first root is significant but the second is not. The loadings indicate that the first root is largely a matter of YEARS ($r = .877$), and the second root a matter of SERIOUS ($r = .998$), but not independent of YEARS ($r = .480$). On page 8 of the output you can find the interaction plot and on page 9 the tests of simple effects. When the defendant was unattractive, the "long sentence despite not serious crime" canonical variate was significantly higher when the crime was a swindle than when it was a burglary. The simple effects of type of crime fell short of significance for defendant of average or high physical attractiveness.

**Main Effects**

The MANOVA output for the main effect of CRIME (pages 4 & 5) indicates no significant effect. Do note that the one root has canonical coefficients different from those of
either of the roots for the PA X CRIME multivariate analysis. That is, the one multivariate composite variable analyzed at this level differs from the two analyzed at the interaction level. For this reason, it might well make sense to interpret a significant multivariate main effect even if that IV did participate in a higher order interaction, because the canonical variates used at the two different levels of analysis are not the same. For example, the effect of PA might differ across levels of Crime for one or both of the canonical variates that maximize the interaction, and Crime might have a significant main effect upon the different canonical variate that maximizes separation of the two Crime groups, and PA might have a significant main effect on one or both of the yet different canonical variates that separate the PA groups. What you must remember is that for each effect, MANOVA computes the one or more sets of weights that maximize that effect, and different effects are maximized by different sets of weights.

One way to avoid having the various effects in your factorial analysis done on different sets of canonical variates is to adopt the “Principal Components then ANOVA” strategy. First, use a Principal Components Analysis with Varimax rotation to isolate from your many correlated outcome variables a smaller number of orthogonal components. Compute and output component scores and then do a univariate factorial ANOVA on each component score. Since the components are orthogonal, there is no need to utilize MANOVA.

Now, look at the output for the effect of PA. We do have a significant multivariate effect. Note, however, that neither of the univariate F's is significant. That is, neither Years nor Serious considered alone is significantly affected by PA, but an optimally weighted pair of linear combinations of these to variables is significantly affected by PA. Clearly the MANOVA here is more powerful than ANOVA, and clearly the unsophisticated “look at univariate tests” strategy for interpreting significant multivariate effects would only confuse its unsophisticated user.

Do note that dimension reduction analysis indicates that we need interpret only the first root. The standardized canonical coefficients and the loadings indicate that this first canonical variate is a matter of Years (length of sentence given the defendant), with a negatively weighted contribution from Seriousness. That is, if the mock juror thought the crime not very serious, but he gave a long sentence anyhow, he gets a high score on this canonical variate. If he gives a light sentence despite thinking the crime serious, he gets a low canonical score. Low scores seem to represent giving the defendant a break for some reason.

I computed the canonical variate scores for the first root of the significant multivariate effect of PA (CV_PA1) and then conducted an ANOVA on the canonical variate, with pairwise comparisons. On page 9 we see that 8.859/108 = .082 = the eigenvalue from page 3. The test of the main effect of PA on this canonical variate, $F(2, 108) = 4.43, p = .0142$, is identical to the test of Roy’s greatest root (page 7). On page 10 we see that the group centroids on this canonical variate are -.21 for the beautiful defendants, -.16 for the average, and +.38 for the unattractive. Fisher’s procedure indicates that the mean for the unattractive defendant is significantly higher than the other two, which do not differ significantly from each other. Keeping in mind the weighting coefficients used to compute CV_PA1, this means that relative to the other two groups, unattractive defendants received sentences that were long even if the mock juror thought the crime was not especially serious.
An ANCOV

The analysis so far suggests that unattractive people seem to get an extra penalty, above and beyond that which is appropriate for the perceived seriousness of their crime. Accordingly, I decided to use ANCOV to see if the groups differed on the sentence recommended after the sentence was statistically adjusted for the perceived seriousness of the crime. The ANCOV indicates that when adjusted for perceived seriousness of the crime, the unattractive defendant received recommended sentences significantly longer than those for defendants of average or great attractiveness.

SPSS

Obtain from my SPSS Programs page MANOVA2.sps. Run this program. The output has essentially the same statistics as the SAS output, with the addition of Box's $M$. I did include one option not requested the last time we used SPSS to do MANOVA -- a Roy-Bargman Stepdown test. This analysis is sequential, with each dependent variable adjusted for the effects of other dependent variables that precede it in the MANOVA statement. I specified SERIOUS before YEARS. Accordingly, the $F$ for YEARS would be for an ANCOV with SERIOUS the covariate. Note that that $F$ is significant at the .015 level, and is absolutely equivalent to the ANCOV we conducted earlier with SAS.

Effect Size Estimates

As in one-way MANOVA, eta-squared can be computed as $\eta^2 = 1 - \Lambda$. For our data, eta-squared is .050 for the interaction, .005 for the main effect of type of crime, and .085 for the main effect of physical attractiveness.

In factorial MANOVA the values of the effects' eta-squared can sum to more than 1, since each effect involves different linear combinations of the outcome variables. Multivariate eta-squared also tends to run larger than univariate eta-squared. One attempt to correct for this is the multivariate partial eta-squared, $\eta^2_p = 1 - \Lambda^{1/s}$, where $s$ is the smaller of number of outcome variables and degrees of freedom for the effect. For our data, partial eta-squared is .024 for the interaction, .005 for the main effect of type of crime, and .043 for the main effect of physical attractiveness.