**Comparing Two Groups’ Factor Structures:
Pearson *r* and the Coefficient of Congruence**

 One may use a simple Pearson *r* to compare a factor in one group with a factor in a second group. Just correlate the loadings on the factor in Group 1 with the loadings on the maybe similar factor in Group 2. The Pearson *r* can detect not only differences in two factors’ patterns of loadings, but also differences in the relative magnitudes of those loadings. One should beware that with factors having a large number of small loadings, those small loadings could cause the *r* to be large (if they are similar between factors) even if the factors had dissimilar loadings on the more important variables.

 Here is an example of the use of *r* to compare the factor structure in two groups. The variables are the 18 items in Allen and Meyer’s (1990) scale of organizational commitment. Three subscales are commonly recognized:

* Continuance commitment (CC), which is the fear of losing “side-bets” one has accrued while working for an organization and a perceived lack of alternatives to the current employment situation
* Affective commitment (AC), which is when one is emotionally attached to and identifies with the organization
* Normative commitment (NC), which is when one feels morally obligated to stay with the organization

 For pedagogical purposes, I split a data set into two groups. Group 0 includes all those who reported their annual income to be less than $60,000. Group 1 includes all those who reported their annual income to be at least $60,000.

|  |
| --- |
| **Income60G** |
|  | Frequency | Percent | Valid Percent | Cumulative Percent |
| Valid | .00 | 102 | 54.3 | 54.5 | 54.5 |
| 1.00 | 85 | 45.2 | 45.5 | 100.0 |
| Total | 187 | 99.5 | 100.0 |  |
| Missing | System | 1 | .5 |  |  |
| Total | 188 | 100.0 |  |  |

I then conducted a principal axis factor analysis with a varimax rotation separately for the two groups. On the next two pages are the loadings for both groups. Next we need to pair up the factors, one from each group, in terms of similarity. For example, it may be that Factor 1 in Group 1 looks very similar to Factor 1 in Group 2 but not very similar to Factor 2 or Factor 3 in Group 2 – but it could be that Factor 1 in Group 1 looks more like Factor 2 in Group 2 than it does like Factor 1 or Factor 3 in Group 2.

 For these data, it is pretty obvious that Factor 1 Lo is similar to Factor 1 Hi, Factor 2 Lo is similar to Factor 3 Hi, and Factor 3 Lo is similar to Factor 2 Hi.

|  |  |
| --- | --- |
| **Income60G = 1.00** | Factor |
| 1 | 2 | 3 |
| OrgCom4 | .885 | -.094 | .147 |
| OrgCom6 | .800 | .029 | .253 |
| OrgCom5 | .786 | -.250 | .241 |
| OrgCom3 | .776 | -.104 | .238 |
| OrgCom16 | .738 | -.117 | .480 |
| OrgCom18 | .721 | -.012 | .279 |
| OrgCom17 | .720 | .021 | .573 |
| OrgCom1 | .694 | -.062 | .133 |
| OrgCom13 | .639 | -.023 | .378 |
| OrgCom2 | .518 | -.092 | .012 |
| OrgCom10 | -.216 | .815 | -.062 |
| OrgCom9 | .216 | .714 | .115 |
| OrgCom8 | .221 | .655 | .111 |
| OrgCom12 | -.141 | .633 | -.136 |
| OrgCom7 | -.243 | .627 | .095 |
| OrgCom11 | -.177 | .535 | -.050 |
| OrgCom15 | .308 | .005 | .853 |
| OrgCom14 | .461 | .108 | .676 |

|  |  |
| --- | --- |
| **Income60G = .00** | Factor |
| 1 | 2 | 3 |
| OrgCom3 | .825 | .179 | -.183 |
| OrgCom4 | .789 | .264 | -.119 |
| OrgCom5 | .784 | .046 | -.249 |
| OrgCom6 | .649 | .409 | -.060 |
| OrgCom1 | .588 | .287 | .199 |
| OrgCom13 | .553 | .477 | .062 |
| OrgCom2 | .493 | .338 | .220 |
| OrgCom15 | .192 | .817 | .200 |
| OrgCom14 | .181 | .715 | .173 |
| OrgCom16 | .500 | .678 | .001 |
| OrgCom17 | .491 | .640 | .098 |
| OrgCom18 | .499 | .540 | -.061 |
| OrgCom10 | -.190 | -.020 | .814 |
| OrgCom12 | -.026 | -.057 | .652 |
| OrgCom8 | .096 | .303 | .644 |
| OrgCom9 | .083 | .313 | .603 |
| OrgCom11 | .017 | -.018 | .459 |
| OrgCom7 | -.195 | .127 | .366 |

Here are the loadings not sorted by size. I copied these unsorted loadings into an SPSS save file.

|  |  |  |
| --- | --- | --- |
| **Income60G = .00** |  | **Income60G = 1.00** |
| Variable | Factor |  | Factor |
| 1 | 2 | 3 |  | 1 | 2 | 3 |
| OrgCom1 | .588 | .287 | .199 |  | .694 | -.062 | .133 |
| OrgCom2 | .493 | .338 | .220 |  | .518 | -.092 | .012 |
| OrgCom3 | .825 | .179 | -.183 |  | .776 | -.104 | .238 |
| OrgCom4 | .789 | .264 | -.119 |  | .885 | -.094 | .147 |
| OrgCom5 | .784 | .046 | -.249 |  | .786 | -.250 | .241 |
| OrgCom6 | .649 | .409 | -.060 |  | .800 | .029 | .253 |
| OrgCom7 | -.195 | .127 | .366 |  | -.243 | .627 | .095 |
| OrgCom8 | .096 | .303 | .644 |  | .221 | .655 | .111 |
| OrgCom9 | .083 | .313 | .603 |  | .216 | .714 | .115 |
| OrgCom10 | -.190 | -.020 | .814 |  | -.216 | .815 | -.062 |
| OrgCom11 | .017 | -.018 | .459 |  | -.177 | .535 | -.050 |
| OrgCom12 | -.026 | -.057 | .652 |  | -.141 | .633 | -.136 |
| OrgCom13 | .553 | .477 | .062 |  | .639 | -.023 | .378 |
| OrgCom14 | .181 | .715 | .173 |  | .461 | .108 | .676 |
| OrgCom15 | .192 | .817 | .200 |  | .308 | .005 | .853 |
| OrgCom16 | .500 | .678 | .001 |  | .738 | -.117 | .480 |
| OrgCom17 | .491 | .640 | .098 |  | .720 | .021 | .573 |
| OrgCom18 | .499 | .540 | -.061 |  | .721 | -.012 | .279 |

CORRELATIONS

 /VARIABLES=Factor1Lo Factor2Lo Factor3Lo

 WITH Factor1Hi Factor2Hi Factor3Hi

 /PRINT=TWOTAIL NOSIG

 /MISSING=PAIRWISE.

|  |
| --- |
| **Correlations** |
|  | Factor1Hi | Factor2Hi | Factor3Hi |
| Factor1Lo | Pearson Correlation | .950\*\* | -.894\*\* | .263 |
| Sig. (2-tailed) | .000 | .000 | .293 |
| N | 18 | 18 | 18 |
| Factor2Lo | Pearson Correlation | .491\* | -.487\* | .893\*\* |
| Sig. (2-tailed) | .039 | .041 | .000 |
| N | 18 | 18 | 18 |
| Factor3Lo | Pearson Correlation | -.866\*\* | .922\*\* | -.466 |
| Sig. (2-tailed) | .000 | .000 | .051 |
| N | 18 | 18 | 18 |

**Tucker’s Coefficient of Congruence**

 Multiply each loading in the one group by the corresponding loading in the other group. Sum these products and then divide by the square root of (the sum of squared loadings for the one group times the sum of squared loading for the other group). For our data,

COMPUTE Lo1\_x\_Hi1=Factor1Lo\*Factor1Hi.

EXECUTE.

COMPUTE Lo2\_x\_Hi3=Factor2Lo\*Factor3Hi.

EXECUTE.

DATASET ACTIVATE DataSet1.

SAVE OUTFILE='C:\Users\Vati\Desktop\FS-2groups\FactorStructure-TwoGroups.sav'

 /COMPRESSED.

COMPUTE Lo3\_x\_Hi2=Factor3Lo\*Factor2Hi.

EXECUTE.

COMPUTE SqLoadings1Lo=Factor1Lo \* Factor1Lo.

EXECUTE.

COMPUTE SqLoadings2Lo=Factor2Lo \* Factor2Lo.

EXECUTE.

COMPUTE SqLoadings3Lo=Factor3Lo \* Factor3Lo.

EXECUTE.

COMPUTE SqLoadings1Hi=Factor1Hi \* Factor1Hi.

EXECUTE.

COMPUTE SqLoadings2Hi=Factor2Hi \* Factor2Hi.

EXECUTE.

COMPUTE SqLoadings3Hi=Factor3Hi \* Factor3Hi.

EXECUTE.

DATASET ACTIVATE DataSet1.

SAVE OUTFILE='C:\Users\Vati\Desktop\FS-2groups\FactorStructure-TwoGroups.sav'

 /COMPRESSED.

DESCRIPTIVES VARIABLES=Lo1\_x\_Hi1 Lo2\_x\_Hi3 Lo3\_x\_Hi2 SqLoadings1Lo SqLoadings2Lo SqLoadings3Lo SqLoadings1Hi SqLoadings2Hi SqLoadings3Hi

 /STATISTICS=SUM.

|  |
| --- |
| **Descriptive Statistics** |
|  | N | Sum |
| Lo1\_x\_Hi1 | 18 | 4.84 |
| Lo2\_x\_Hi3 | 18 | 2.53 |
| Lo3\_x\_Hi2 | 18 | 2.48 |
| SqLoadings1Lo | 18 | 4.13 |
| SqLoadings2Lo | 18 | 3.24 |
| SqLoadings3Lo | 18 | 2.50 |
| SqLoadings1Hi | 18 | 5.94 |
| SqLoadings2Hi | 18 | 2.80 |
| SqLoadings3Hi | 18 | 2.24 |
| Valid N (listwise) | 18 |  |

 Multiply each loading in the one group by the corresponding loading in the other group. Sum these products and then divide by the square root of (the sum of squared loadings for the one group times the sum of squared loading for the other group). For our data,







 I should add that I have not followed the usual procedure here. Usually one of the solutions is subjected to a procrustean rotation, which makes it as similar as possible to the other solution. This should, of course, make the coefficients of congruence even greater than they would be without such a rotation. Accordingly, the values I obtained above are conservative.

 So, how large should the coefficient be before you declare the factors highly similar. Lorenzo-Seva and ten Berge (2006) suggested “a value in the range .85–.94 corresponds to a fair similarity, while a value higher than .95 implies that the two factors or components compared can be considered equal.”

**Procrustean Rotation**

 Ronald Fischer taught me how to conduct a procrustean rotation with SPSS – see [his blog](http://culturemindspace.blogspot.co.nz/2012/03/how-to-do-procrustean-factor-rotation.html) . He also provided me with the syntax. From my [SPSS programs page](http://core.ecu.edu/psyc/wuenschk/SPSS/SPSS-Programs.htm), download ProcrusteanRotation.sps. The first matrix contains the loadings from the factor analysis with the first group, and the second matrix has the loadings from the second group. For the first group the loadings are ordered Factor 1, Factor 2, Factor 3. For the second group they are ordered Factor 1, Factor 3, Factor 2. When you run the syntax you get the procrustean rotated loadings for the second group and several indices of factor similarity.

 The next step is to compute Tucker’s coefficient of congruence. Do it just like I did earlier, but replace the second group’s loadings with those obtained by the procrustean rotation. Here are those loadings. The first column is for Factor 1, the second is for Factor 3, and the third is for Factor 2.

FACTOR LOADINGS AFTER TARGET ROTATION

 .65 .14 .17

 .57 .21 .20

 .83 .02 -.24

 .82 .11 -.17

 .76 -.09 -.31

 .71 .27 -.09

 -.14 .13 .39

 .20 .22 .65

 .19 .24 .61

 -.13 -.05 .82

 .04 -.06 .45

 .01 -.10 .65

 .64 .35 .05

 .34 .65 .20

 .37 .74 .24

 .63 .56 .00

 .62 .52 .10

 .60 .43 -.07

 Here are the statistics needed to calculate the coefficient of congruence.

|  |
| --- |
| **Descriptive Statistics** |
|  | N | Sum |
| Lo1\_x\_Hi1 | 18 | 4.54 |
| Lo2\_x\_Hi3 | 18 | 2.60 |
| Lo3\_x\_Hi2 | 18 | 2.55 |
| SqLoadings1Lo | 18 | 4.13 |
| SqLoadings2Lo | 18 | 3.24 |
| SqLoadings3Lo | 18 | 2.50 |
| SqLoadings1Hi | 18 | 5.11 |
| SqLoadings2Hi | 18 | 2.62 |
| SqLoadings3Hi | 18 | 2.16 |
| Valid N (listwise) | 18 |  |







Reference

Lorenzo-Seva, U., & ten Berge, J. M. F. (2006). Tucker’s congruence coefficient as a meaningful index of factor similarity. *Methodology*; *2*(2):57–64. doi 10.1027/1614-1881.2.2.57

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