Mathematics 2173-002
Calculus III
Fall 2012
T, Th 9:00 AM - 10:40 PM, Howell East 205
Test# 2

(1) If \( \vec{u}, \vec{v}, \vec{w} \) are coplanar, then their triple product
(a) is 3.
(b) is 1.
(c) is 0.
(d) does not exist.
(e) None of the above

(2) What is the formula for computing \( \vec{a} \cdot (\vec{b} \times \vec{c}) \) if \( \vec{a} = (a_1, a_2, a_3), \vec{b} = (b_1, b_2, b_3), \vec{c} = (c_1, c_2, c_3) \)?
(a) This product does not make sense.
(b) \[ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \]
(c) \[ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \]
(d) \[ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} \]
(e) None of the above

(3) Compute the volume of the parallelepiped based on position vectors \((1, 0, 0), (0, 1, 0), (1, 1, 1)\).
(a) 1
(b) 2
(c) 3
(d) 4
(e) None of the above

(4) Suppose an ellipse has the foci at \((-6, -1)\) and \((2, -1)\). Which of the following formulas can be the formula for this ellipse for some values of \(a\) and \(b\) satisfying the stated conditions?
(a) \[ \frac{x - 2}{a^2} + \frac{y + 1}{b^2} = 1, a^2 + b^2 = 16 \]
(b) \[ \frac{x + 4}{a^2} + \frac{y + 1}{b^2} = 1, a^2 - b^2 = 16 \]
(c) \[ \frac{x + 4}{a^2} + \frac{y - 1}{b^2} = 1, a^2 + b^2 = 16 \]
(d) \[ \frac{x - 4}{a^2} + \frac{y - 1}{b^2} = 1, a^2 - b^2 = 16 \]
(e) None of the above

(5) What are the conic sections?
(a) Conic sections are the curves which result from intersecting a plane and a cone.
(b) Conic sections are the curves which are the results of intersection of two planes.
(c) Conic sections are the curves which are the result of intersection of a plane and a circle.
(d) Conic sections are the curves which are the results of intersection of two lines.
(e) None of the above

(6) Which of the statements below is true?
(a) A point \( P \) is on the parabola with foci \( F_1 \) and \( F_2 \) if and only if the absolute value of the difference of the distances from \( P \) to \( F_1 \) and from \( P \) to \( F_2 \) is equal to some fixed (for the parabola) real number \( 2a > 0 \).
(b) A parabola is the set of all the points on the plane equidistant from a line (directrix) and a point (focus).
(c) A point \( P \) is on the parabola with foci \( F_1 \) and \( F_2 \) if and only if the distance from \( P \) to \( F_1 \) plus the distance from \( P \) to \( F_2 \) is equal to some fixed (for the parabola) real number \( 2a > 0 \).
(d) A point \( P \) is on the parabola with a focus \( F \) and a directrix \( \ell \) if and only if the distance from \( P \) to \( F \) plus the distance from \( P \) to \( \ell \) is equal to some fixed (for the parabola) real number \( 2a > 0 \).
(e) None of the above

(7) Which of the statements below is true?
(a) A point \( P \) is on the ellipse with foci \( F_1 \) and \( F_2 \) if and only if the absolute value of the difference of the distances from \( P \) to \( F_1 \) and from \( P \) to \( F_2 \) is equal to some fixed (for the ellipse) real number \( 2a > 0 \).
(b) An ellipse is the set of all the points on the plane equidistant from a line (directrix) and a point (focus).
(c) A point \( P \) is on the ellipse with foci \( F_1 \) and \( F_2 \) if and only if the distance from \( P \) to \( F_1 \) plus the distance from \( P \) to \( F_2 \) is equal to some fixed (for the ellipse) real number \( 2a > 0 \).
(d) A point \( P \) is on the ellipse with a focus \( F \) and a directrix \( \ell \) if and only if the distance from \( P \) to \( F \) plus the distance from \( P \) to \( \ell \) is equal to some fixed (for the ellipse) real number \( 2a > 0 \).
(e) None of the above

(8) Which of the statements below is true?
(a) A point \( P \) is on the hyperbola with a focus \( F \) and a directrix \( \ell \) if and only if the distance from \( P \) to \( F \) plus the distance from \( P \) to \( \ell \) is equal to some fixed (for the hyperbola) real number \( 2a > 0 \).
(b) An hyperbola is the set of all the points on the plane equidistant from a line (directrix) and a point (focus).
(c) A point \( P \) is on the hyperbola with foci \( F_1 \) and \( F_2 \) if and only if the distance from \( P \) to \( F_1 \) plus the distance from \( P \) to \( F_2 \) is equal to some fixed (for the hyperbola) real number \( 2a > 0 \).
(d) A point \( P \) is on the hyperbola with foci \( F_1 \) and \( F_2 \) if and only if the absolute value of the difference of the distances from \( P \) to \( F_1 \) and from \( P \) to \( F_2 \) is equal to some fixed (for the hyperbola) real number \( 2a > 0 \).
(e) None of the above
(9) Which of the following statements is true:
(a) An ellipse with a directrix \( x = -p \) and a foci \((p, 0)\) has an equation \( y^2 = 4px \).
(b) An ellipse with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), where \(a \geq b > 0\) and \(c^2 = a^2 - b^2\).
(c) An ellipse with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \), where \(c^2 = a^2 + b^2\), and the asymptotes are \( y = \pm \frac{b}{a}x \).
(d) An ellipse with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), where \(a \geq b > 0\) and \(c^2 = a^2 + b^2\).
(e) None of the above

(10) Which of the following statements is true:
(a) A hyperbola with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), where \(a \geq b > 0\) and \(c^2 = a^2 + b^2\).
(b) A hyperbola with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), where \(a \geq b > 0\) and \(c^2 = a^2 - b^2\).
(c) A hyperbola with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \), where \(c^2 = a^2 + b^2\), and the asymptotes are \( y = \pm \frac{b}{a}x \).
(d) A hyperbola with a directrix \( x = -p \) and a foci \((p, 0)\) has an equation \( y^2 = 4px \).
(e) None of the above

(11) Which of the following statements is true:
(a) A parabola with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), where \(a \geq b > 0\) and \(c^2 = a^2 - b^2\).
(b) A parabola with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \), where \(c^2 = a^2 + b^2\), and the asymptotes are \( y = \pm \frac{b}{a}x \).
(c) A parabola with a directrix \( x = -p \) and a foci \((p, 0)\) has an equation \( y^2 = 4px \).
(d) A parabola with vertices at \((\pm a, 0)\) and foci \((\pm c, 0)\) has an equation of the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), where \(a \geq b > 0\) and \(c^2 = a^2 + b^2\).
(e) None of the above

(12) Find all the foci of the conic section \( \frac{x^2}{25} + \frac{y^2}{16} = 1 \).
(a) This conic section has no foci.
(b) \((0, -3)\)
(c) \((0, 3)\)
(d) \((\pm 3, 0)\)
(e) None of the above

(13) Find all the foci of the conic section \( \frac{(x - 3)^2}{25} + \frac{(y - 1)^2}{16} = 1 \).
(a) This conic section has no foci.
(b) \((1, -3), (1, 3)\)
(c) (0, 1), (6, 1)
(d) (±1, 4)
(e) None of the above

(14) Find all the foci of the conic section \( \frac{y^2}{16} - \frac{x^2}{9} = 1 \).
(a) This conic section has no foci.
(b) (0, \(-\sqrt{7}\))
(c) (±\sqrt{7}, 0)
(d) (0, ±5)
(e) None of the above

(15) Find all the foci of the conic section \( x^2 = 16y \).
(a) This conic section has no foci.
(b) (0, 4)
(c) (0, ±4)
(d) None of the above

(16) Find all the foci of the conic section \( (x - 2)^2 = 8y - 8 \).
(a) This conic section has no foci.
(b) (0, 2)
(c) (2, 8)
(d) (2, ±4)
(e) None of the above

(17) Identify the graph of the following equation: \( 4x^2 + x - y^2 + y = 0 \).
(a) This is an ellipse which is not a circle.
(b) This is a hyperbola.
(c) This is a parabola
(d) This is a circle.
(e) None of the above

(18) If \((x_1, y_1, z_1), (x_2, y_2, z_2)\) are two points in \(\mathbb{R}^3\), then the distance \(d\) between these points can be computed using the following formula:
(a) \( d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \)
(b) \( d = \sqrt{(x_1 + x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \)
(c) \( d = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2 + (z_1 + z_2)^2} \)
(d) \( d = \sqrt{(x_1 - x_2)^2 + (y_1 + y_2)^2 + (z_1 - z_2)^2} \)
(e) None of the above

(19) Which of the following equations corresponds to a sphere of radius 3 centered at \((1, -2, 3)\)?
(a) \( (x - 1)^2 + (y + 2)^2 + (z - 3)^2 = 9 \)
(b) \( (x + 1)^2 + (y + 2)^2 + (z - 3)^2 = 4 \)
(c) \( (x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 3 \)
(d) \( (x + 1)^2 + (y + 2)^2 + (z - 3)^2 = 2 \)
(e) None of the above

(20) Which of the following statements is not true:
(a) The direction of \(\vec{0}\) is undefined.
(b) Vector addition is commutative.
(c) If two vectors have the same direction, they have the same magnitude (or length).
(d) If $k < 0$, then $k\vec{v}$ and $\vec{v}$ have opposite direction.
(e) None of the above

(21) If $A, B, C$ are three non-collinear points, then which of the following statements is true for any such points? (Hint: draw the vectors.)
(a) $\vec{AB} - \vec{BC} = \vec{CA}$
(b) $\vec{AB} + \vec{BC} = \vec{BA}$
(c) $\vec{BA} + \vec{AC} = \vec{BC}$
(d) $\vec{AB} + \vec{BC} = \vec{BA}$
(e) None of the above

(22) If $A, B, C$ are three non-collinear points, then which of the following statements is true? (Hint: draw the vectors.)
(a) $\vec{AB} + \vec{AC} = \vec{CB}$
(b) $\vec{AB} - \vec{AC} = \vec{BC}$
(c) $\vec{AB} + \vec{AC} = \vec{BC}$
(d) $\vec{BA} - \vec{BC} = \vec{CA}$
(e) None of the above

(23) If $A, B, C, D$ are vertices of a parallelogram in order, then which of the following statements is true? (Hint: draw the vectors.)
(a) $\vec{DA} + \vec{DC} = \vec{DB}$
(b) $\vec{AB} - \vec{AD} = \vec{CA}$
(c) $\vec{AB} + \vec{AD} = \vec{CA}$
(d) $\vec{AB} - \vec{AD} = \vec{CA}$
(e) None of the above

(24) If $\vec{u} = (1, 2, 3), \vec{v} = (0, 1, 0)$, then what are the coordinates of $\vec{u} + 3\vec{v}$?
(a) $(1, 7, 3)$
(b) $(1, 5, 3)$
(c) $(1, 7, -6)$
(d) $(2, -7, 6)$
(e) None of the above

(25) What are the coordinates of the standard basis vectors in $\mathbb{R}^3$?
(a) $\vec{i} = (1, 0, 0), \vec{j} = (1, 1, 0), \vec{k} = (1, 1, 1)$
(b) $\vec{i} = (1, 1, 0), \vec{j} = (0, 1, 1), \vec{k} = (1, 0, 1)$
(c) $\vec{i} = (1, 0, 1), \vec{j} = (0, 1, 0), \vec{k} = (0, 0, 1)$
(d) $\vec{i} = (1, 0, 0), \vec{j} = (0, 2, 0), \vec{k} = (0, 0, 3)$
(e) None of the above

(26) What is a position vector?
(a) A vector whose position is unknown
(b) A vector with tip/end point at the origin
(c) A vector with its middle point at the origin.
(d) A vector with its tail/starting point at the origin
(e) None of the above

(27) What are the coordinates of the position vector equal to vector $\vec{AB}$, if $A$ has coordinates $(4, 4, 4)$ and $B$ has coordinates $(3, 3, 3)$?
(a) \((-1, -1, -1)\)
(b) \((2, 2, 2)\)
(c) \((3, 3, 3)\)
(d) \((4, 4, 4)\)
(e) None of the above

(28) What is the magnitude of a position vector \(\vec{u}\) with coordinates \((0, 3, 4)\)?
(a) \(\sqrt{2}\)
(b) 5
(c) \(\sqrt{5}\)
(d) 7
(e) None of the above

(29) What are the coordinates of a unit vector in the same direction as a position vector \(\vec{u}\) with coordinates \((0, 3, 4)\)?
(a) \(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\)
(b) \(\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}\)
(c) \(0, \frac{3}{\sqrt{5}}, 1\)
(d) \(0, \frac{3}{5}, 4\)
(e) None of the above

(30) Write \(\vec{v}\) with coordinates \((1, 4, 3)\) as a sum of standard basis vectors.
(a) \(2\vec{i} + \vec{j} + 5\vec{k}\)
(b) \(\vec{i} + 4\vec{j} + 3\vec{k}\)
(c) \(\vec{i} - 4\vec{j} + 3\vec{k}\)
(d) \(\vec{i} + 4\vec{j} + \vec{k}\)
(e) None of the above

(31) If \(\vec{u} = (a_1, a_2, a_3)\) and \(\vec{v} = (b_1, b_2, b_3)\), then the dot product \(\vec{u} \cdot \vec{v}\) is
(a) a number equal to \(a_1b_1 + a_2b_2 + a_3b_3\)
(b) a number equal to \(a_1b_1 - a_2b_2 + a_3b_3\)
(c) a vector of length \(a_1b_1 + a_2b_2 + a_3b_3\)
(d) not always defined.
(e) None of the above

(32) If \(\vec{u} = (1, 2, 3)\) and \(\vec{v} = (8, 10, 12)\), then the dot product \(\vec{u} \cdot \vec{v}\) is
(a) 64
(b) 0
(c) −64
(d) is not defined.
(e) None of the above

(33) Which of the following statements is true for all vectors \(\vec{u}, \vec{v}\) and all real numbers \(c\)?
(a) \(\vec{u} \cdot \vec{v} = -\vec{v} \cdot \vec{u}\)
(b) \(\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{v} \cdot \vec{u} - \vec{v} \cdot \vec{w}\)
(c) \(c\vec{u} \cdot \vec{v} = -(c\vec{v}) \cdot \vec{u}\)
(d) \(\vec{u} \cdot \vec{v} = -\vec{v} \cdot 2\vec{u}\)
(e) None of the above

(34) Which of the following statements is true for all vectors \(\vec{u}, \vec{v}\) and all real numbers \(c\)?
(a) $\bar{u} \cdot \bar{u} = |\bar{u}|$
(b) $\bar{u} \cdot \bar{u} = |\bar{u}|^{1/2}$
(c) $\bar{u} \cdot \bar{0} = \bar{u}$ (here $\bar{0}$ is the zero vector)
(d) $c\bar{u} \cdot \bar{v} = -\bar{v} - c\bar{u}$
(e) None of the above

(35) If $\alpha$ is the angle between vectors $\bar{u}$ and $\bar{v}$, then $\bar{u} \cdot \bar{v}$ is equal to
(a) $|\bar{u}| |\bar{v}| \sin \alpha$
(b) $|\bar{u}| |\bar{v}| \tan \alpha$
(c) $|\bar{u}| |\bar{v}| \sec \alpha$
(d) $|\bar{u}| |\bar{v}| \cos \alpha$
(e) None of the above

(36) If the dot product of two non-zero vectors is equal to zero then
(a) the vectors are perpendicular.
(b) the vectors are parallel.
(c) the vectors have the same length.
(d) the vectors must have opposite direction.
(e) None of the above

(37) Which of the vectors below are perpendicular to each other:
(a) $(1, 2, 1)$ and $(3, 1, 1)$
(b) $(2, 0, 12)$ and $(1, 0, 5)$
(c) $(0, 15, 0)$ and $(23, 0, 2)$
(d) $(1, 1, 4)$ and $(-1, -1, -1)$
(e) None of the above

(38) If $|\bar{u}| = 1$, $|\bar{v}| = 1$ and the angle between $\bar{u}$ and $\bar{v}$ is $\pi/6$, then $\bar{u} \cdot \bar{v}$ is equal to
(a) $\sqrt{3}/2$
(b) $1/2$
(c) $0$
(d) $\sqrt{2}$
(e) None of the above

(39) If the dot product of two unit vectors is $-1$, then the angle between the vectors
(a) is $\pi$.
(b) cannot be determined.
(c) is $0$.
(d) is $\pi/2$.
(e) None of the above

(40) Which of the following expressions is not equal to the scalar projection of $\bar{u}$ onto $\bar{v} \neq 0$ if the angle between the vectors is $\theta$:
(a) $|\bar{u}| \sec \theta$
(b) $|\bar{u}| \cos \theta$
(c) $\bar{u} \cdot \bar{v}$
(d) $\frac{\bar{u} \cdot \bar{v}}{|\bar{v}|}$
(e) None of the above

(41) Let $\bar{u} = (1, 0, 1)$, let $\bar{v} = (2, 2, 0)$ and compute the scalar projection of $\bar{u}$ onto $\bar{v}$. 

(a) $\frac{1}{\sqrt{2}}$
(b) $-\sqrt{3}$
(c) $\sqrt{3}$
(d) $-\frac{1}{\sqrt{2}}$
(e) None of the above

(42) The vector projection of a vector $\vec{u}$ onto a non-zero vector $\vec{v}$
(a) is a number equal to minus the scalar projection of $\vec{u}$ onto $\vec{v}$.
(b) does not always exist.
(c) is a vector equal to the product of a unit vector in the direction of $\vec{u}$ and the scalar projection of $\vec{v}$ onto $\vec{u}$.
(d) is a vector equal to the product of a unit vector in the direction of $\vec{v}$ and the scalar projection of $\vec{u}$ onto $\vec{v}$.
(e) None of the above

(43) Let $\vec{u} = (1, 0, 1)$, let $\vec{v} = (2, 2, 0)$ and compute the vector projection of $\vec{u}$ onto $\vec{v}$.
(a) $(0, 0, 1)$
(b) $\left(\frac{1}{3}, \frac{1}{3}, 0\right)$
(c) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
(d) $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$
(e) None of the above

(44) The result of the cross product of two non-zero vectors is
(a) a number.
(b) is a vector parallel to one of the given vectors.
(c) is a vector co-planar with the the two given vectors.
(d) is a vector perpendicular to both given vectors.
(e) None of the above

(45) If $\vec{u} = (a_1, a_2, a_3)$ and $\vec{v} = (b_1, b_2, b_3)$, then the cross product $\vec{u} \times \vec{v}$ is equal to
(a) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(b) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$

(c) $\begin{vmatrix} \hat{k} & \hat{j} & \hat{i} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(d) $\begin{vmatrix} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(e) None of the above

(46) Compute $(1, 1, 0) \times (2, 2, 0)$. 
(a) (−1, −1, 1)
(b) (1, −1, −1)
(c) (1, −1, 1)
(d) (−1, −1, −1)
(e) None of the above

(47) If |\vec{u}| = 2, |\vec{v}| = 4 and the angle between the vectors is 30°, then |\vec{u} \times \vec{v}| =
(a) 1
(b) 2
(c) 3
(d) 4
(e) None of the above

(48) If \vec{u} is parallel to \vec{v}, then \vec{u} \times \vec{v} =
(a) -1
(b) 0
(c) 1
(d) 2
(e) None of the above