A common inhabitant of human intestines is the bacterium *Escherichia coli*. A cell of this bacterium in a nutrient-broth medium divides into two cells every 20 minutes. The initial population of a culture is 75 cells.

\[ P = P_0 e^{kt} \]

is measured in hours

\[ \ln 2 = \frac{k}{3} \]

\[ k = 3 \ln 2 = \ln 2^3 = \ln 8 \]

\[ P = 75 e^{(3\ln 2)t} \]

\[ 2 = e^{\ln 8} \]

\[ 2 \ln (20,000) = \ln (75) + t \ln 8 \]

\[ t = \frac{\ln (20,000) - \ln (75)}{\ln 8} \]

\[
\begin{align*}
\text{Rate of growth: } \frac{dP}{dt} &= 75 (\ln 8) e^{(\ln 8)t} \\
\text{or } \frac{dP}{dt} &= 75 e^{(3\ln 2)t} e^{(\ln 8)t} \\
\text{or } \frac{dP}{dt} &= 75 e^{(3\ln 2 + \ln 8)t} \\
\end{align*}
\]
A roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F. (Round your answers to the nearest whole number.)

(a) If the temperature of the turkey is 150°F after half an hour, what is the temperature after 50 minutes?

Newton’s Law of Cooling

\[
\frac{dT}{dt} = k (T - T_a)
\]

\[
y = Ce^{kt}
\]

\[
T = C e^{kt} + T_a
\]

\[
T_0 = 185^\circ F
\]

\[
T_a = 75^\circ F
\]

\[
T(0.5) = 150
\]

\[
185 = C e^{k0.5} + 75
\]

\[
185 = C + 75
\]

\[
C = 110
\]

\[
75 = 110 e^{0.5k}
\]

\[
\frac{75}{110} = e^{0.5k}
\]

\[
\ln\left(\frac{15}{22}\right) = 0.5k
\]

\[
k = 2 \ln\left(\frac{15}{22}\right)
\]
Scientist can determine the age of ancient objects by a method called radiocarbon dating. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon, $^{14}\text{C}$, with a half-life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates $^{14}\text{C}$ through food chains. When a plant or animal dies, it stops replacing its carbon and the amount of $^{14}\text{C}$ begins to decrease through radioactive decay. Therefore, the level of radioactivity must also decay exponentially.

A parchment fragment was discovered that had about 68% as much $^{14}\text{C}$ radioactivity as does plant material on Earth today. Estimate the age of the parchment. (Round your answer to the nearest hundred years.)

$$y = y_0 e^{kt}, \quad k < 0$$

$$\frac{1}{2}y_0 = y_0 e^{5730k} \quad \frac{1}{2} = e^{5730k}$$

$$y = y_0 e^{-\left(\frac{\ln 2}{5730}\right)t} \quad \ln \frac{1}{2} = 5730 \cdot k$$

$$0.68 y_0 = y_0 e^{-\left(\frac{\ln 2}{5730}\right)t} \quad k = -\frac{\ln 2}{5730}$$

$$0.68 = e^{-\left(\frac{\ln 2}{5730}\right)t}$$

$$\ln (0.68) = -\frac{\ln 2}{5730} \cdot t$$

$$t = -\frac{5730 \ln (0.68)}{\ln 2}$$
(a) If $1500 is borrowed at 9% interest, find the amounts due at the end of 3 years if the interest is compounded as follows. (Round your answers to the nearest cent.)

\[ A_0 = 1500 \]

\[ r = 0.09 \text{ (annual)} \]

\[ n \text{ compounding periods during the year} \]

\[ A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt} \quad \text{as } n \to \infty \]

\[ A(t) = A_0 e^{rt} \]

annually: \( n = 1 \)

\[ A(3) = 1500 \left(1 + \frac{0.09}{1}\right)^3 = 1500 (1.09)^3 \]
One more application of inverse trig func.

\[\frac{-\pi}{2} \leq u \leq \frac{\pi}{2}\]
\[u = \arcsin (2x)\]
\[\sin u = 2x\]
\[\cos u \, du = 2 \, dx\]
\[dx = \frac{\cos u \, du}{2}\]

\[
\sqrt{1-4x^2} = \sqrt{1-\sin^2 u} = |\cos (u)|
\]

\[
\int \sqrt{1-4x^2} \, dx = \int \cos (u) \cos (\frac{u}{2}) \, du = \frac{1}{2}\int \cos^2 (u) \, du
\]

\[1-4x^2 \geq 0\]
\[4x^2 \leq 1\]
\[|2x| \leq 1\]
\[-1 \leq 2x \leq 1\]
$$\lim_{x \to \infty} \frac{\sin x}{x} = 0$$

$$\frac{\cos x}{1} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$$g(x) \leq f(x) \leq h(x)$$

$\downarrow$  $\downarrow$  $\downarrow$

$\leftarrow$  $\leftarrow$  $\leftarrow$

$L$  $L$  $L$
L'Hospital's Rule (Section 6.8)

\[
\lim_{{x \to c}} \frac{f(x)}{g(x)} = \lim_{{x \to c}} \frac{f'(x)}{g'(x)}
\]

C can be a real number or \( c = \pm \infty \) and all the limits can be one-sided.
Example 1

\[\lim_{x \to 1} \frac{\ln x}{x-1} = \lim_{x \to 1} \frac{1}{x} = \lim_{x \to 1} \frac{1}{x} = 1\]

\[\lim_{x \to 1} \ln x = \ln 1 = 0\]

\[\lim_{x \to 1} (x-1) = 0\]
Example 2

\[
\lim_{x \to \infty} \frac{e^x}{x^2} = \lim_{x \to \infty} \frac{e^x}{2x} = \lim_{x \to \infty} \frac{e^x}{2} = \infty
\]

\[
\lim_{x \to \infty} \frac{e^x}{x^n} = \lim_{x \to \infty} \frac{e^x}{nx^{n-1}} = \lim_{x \to \infty} \frac{e^x}{n(n-1)x^{n-2}}
\]

\[
\cdots \lim_{x \to \infty} \frac{e^x}{h!} = \infty
\]
Example 3

\[
\lim_{x \to \infty} \frac{\ln x}{x^{\frac{1}{3}}} = \lim_{x \to \infty} \frac{1}{\frac{1}{3} x^{- \frac{2}{3}}} = 3 \lim_{x \to \infty} \frac{x^{\frac{2}{3}}}{x} =
\]

\[
= 3 \lim_{x \to \infty} \frac{1}{x^{\frac{1}{3}}} = \lim_{x \to \infty} \frac{\ln x}{x^{2-1}} = \lim_{x \to \infty} \frac{1}{x^{2}} = 0
\]
Example 4

\[ \lim_{x \to 0} \frac{\tan x - x}{x^3} = \lim_{x \to 0} \frac{\sec^2 x - 1}{3x^2} = \frac{2\sec^4 x}{3} \]

\[ \lim_{x \to 0} \frac{2\sec x \cdot \sec x \cdot \tan x}{6x} = \lim_{x \to 0} \frac{4 \sec x \cdot \sec x \cdot \tan x}{6} = \frac{2}{6} = \frac{1}{3} \]
Example 6

\[
\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{1}{x^2} = 0
\]

Example 7

\[
\lim_{x \to \infty} xe^x = \infty
\]

\[
\lim_{x \to -\infty} xe^x = \lim_{x \to -\infty} \frac{x}{e^{-x}} = 0
\]

\[
\lim_{x \to -\infty} e^x = 0
\]

\[
\lim_{x \to -\infty} e^{-x} = 0
\]
Example 8

\[ \lim_{{x \to \frac{\pi}{2}^-}} (\sec x - \tan x) = \lim_{{x \to \frac{\pi}{2}^-}} \left( \frac{1}{\cos x} - \frac{\sin x}{\cos x} \right) \]

\[ = \lim_{{x \to \frac{\pi}{2}^-}} \frac{1 - \sin x}{\cos x} = \lim_{{x \to \frac{\pi}{2}^-}} \frac{-\cos x}{-\sin x} = 0 \]

Example 9

\[ \lim_{{x \to 0^+}} (1 + \sin 4x) \]

\[ = \lim_{{x \to 0^+}} \frac{\cot x}{1 + \sin 4x} = L \]

\[ \ln L = \lim_{{x \to 0^+}} \ln (1 + \sin 4x) = \lim_{{x \to 0^+}} \frac{\ln(1 + \sin 4x)}{\tan x} \]

\[ \lim_{{x \to 0^+}} \frac{4 \cos 4x}{1 + \sin 4x} = \lim_{{x \to 0^+}} \frac{4 \cos 4x}{\cos^2 x} = 4 \]