(1) If \( r : \mathbb{R} \to \mathbb{R}^4 \) is a vector function, then its range consists of
(a) real numbers.
(b) pairs of real numbers.
(c) triples of real numbers.
(d) quadruples of real numbers.
(e) None of the above

(2) If \( \vec{r}(t) = (t^2 - 1 - \cos t, t + 2) \), then \( \lim_{t \to 0} \vec{r}(t) = \)
(a) \((1, 1, 2)\)
(b) does not exist.
(c) \((3, 2, 1)\)
(d) \((1, 3, 2)\)
(e) None of the above

(3) If \( \vec{r}(t) = (t^2 + 1, \sin(t^2), e^t \cos t) \), then \( \vec{r}'(t) \) is
(a) continuous at \( t=0 \).
(b) is not continuous at 0 because the second component is not continuous at \( t=0 \).
(c) is not continuous at 0 because the first component is not continuous at \( t=0 \).
(d) is not continuous at 0 because the third component is not continuous at \( t=0 \).
(e) None of the above

(4) If \( \vec{r}(t) = f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k} \), where \( f(t), g(t), h(t) \) are differentiable functions, then \( \vec{r}'(t) \)
(a) does not always exist.
(b) is equal to \( h'(t) \).
(c) is equal to \( f'(t)\hat{i} - g'(t)\hat{j} + h'(t)\hat{k} \).
(d) is equal to \( -f'(t)\hat{i} - g'(t)\hat{j} - h'(t)\hat{k} \).
(e) None of the above

(5) If \( \vec{r}(t) = (t, \sin(t^2), e^t \cos t) \), then \( \vec{r}'(t) \)
(a) does not exist.
(b) cannot be determined from the given data.
(c) is equal to \( \hat{i} + 2t \cos t\hat{j} + (e^t \cos t - e^t \sin t)\hat{k} \).
(d) is equal to \( \sin t\hat{j} + e^t \cos t\hat{k} \).
(e) None of the above

(6) If \( \vec{r}(t) = (t, t^2, e^t) \), then the unit tangent vector of the space curve \( \vec{r}(t) \), as a function of \( t \), is defined by the following formula.
(a) \((1, 2t, e^t)\)
(b) \( \frac{1}{1 + 4t^2 + e^{2t}}(1, 2t, e^t) \)
(c) \( \frac{1}{\sqrt{4t^2 + e^{2t}}}(1, 2t, e^t) \)
(d) \( \frac{1}{\sqrt{1 + 4t^2 + e^{2t}}} (1, 2t, e^t) \)

(e) None of the above

(7) If \( \vec{r}(t) = (t + 1, t^2, e^t) \), then the tangent line to the curve \( \vec{r}(t) \) at the point \((1, 1, 1)\) has the following equation.

(a) \((0, 1, 2) + t(1, 0, 1)\), where \(t\) is any real number

(b) \((0, 0, 2) + t(1, 1, 1)\), where \(t\) is any real number

(c) \((1, 1, 1) + t(1, 3, e^t)\), where \(t\) is any real number

(d) \((1, 3, e^t) + t(1, 1, 1)\), where \(t\) is any real number

(e) None of the above

(8) If \( \vec{u}(t), \vec{v}(t) \) are differentiable vector functions, then \((\vec{u}(t) + \vec{v}(t))' \)

(a) does not necessarily exist.

(b) is equal to \(\vec{u}'(t) \cdot \vec{v}'(t)\).

(c) is equal to \(\vec{u}'(t) \cdot \vec{v}(t) + \vec{u}(t) \cdot \vec{v}'(t)\).

(d) is equal to \(\vec{u}'(t) + \vec{v}'(t)\).

(e) None of the above

(9) If \( \vec{r}(t) = (t + 2, t^2 + 2, 2e^t + 2) \), then \( \int_0^1 \vec{r}(t)\,dt \)

(a) is equal to \((\frac{1}{2}, \frac{1}{3}, 2)\).

(b) is equal to \((\frac{3}{2}, \frac{7}{2}, 2e - 1)\).

(c) does not exist.

(d) is equal to \((\frac{5}{3}, 2e)\).

(e) None of the above

(10) If \( \vec{r}(t) \) is a space curve such that \( \vec{r}'(t) \) is continuous and every point on the curve is traversed exactly once as \(t\) increases from \(a \in \mathbb{R}\) to \(b \in \mathbb{R}\), then the length of the curve between the point \(\vec{r}(a)\) and \(\vec{r}(b)\) can be calculated using

(a) \( \int_a^b |\vec{r}'(t)|\,dt \)

(b) \( \int_a^b |\vec{r}(t)|\,dt \)

(c) \( \int_a^b |\vec{r}'(t)|\,dt \)

(d) \( \int_a^b |\vec{r}'(t)|\,dt \)

(e) None of the above

(11) Let \( \vec{r}(t) = (2\cos(t + 1) + 2)i + (2\sin(t + 1) - 1)j + (t + 3)k \). Compute the length of the arc from the point \((4, -1, 2)\) to \((\sqrt{2} + 2, \sqrt{2} - 1, \pi/4 + 2)\).

(a) \( \frac{3\pi}{\sqrt{2}} \)

(b) \( \frac{\pi}{4} \)

(c) \( \frac{\sqrt{5}\pi}{4} \)
(d) \( \frac{7\pi}{4} \)
(e) None of the above

(12) Reparametrize the curve \( \vec{r}(t) = (2\cos(t + 1) + 2)\hat{i} + (2\sin(t + 1) - 1)\hat{j} + (t + 3)\hat{k} \) using arc length as measured from the point \((4, -1, 2)\).
(a) \( \vec{r}(t(s)) = (2\cos(s/3))\hat{i} + (2\sin(s/3))\hat{j} + \frac{s}{3}\hat{k} \)
(b) \( \vec{r}(t(s)) = (2\cos(\frac{s}{\sqrt{5}}) + 2)\hat{i} + (2\sin(\frac{s}{\sqrt{5}}) - 1)\hat{j} + (\frac{s}{\sqrt{5}} + 2)\hat{k} \)
(c) \( \vec{r}(t(s)) = (2\cos(\frac{s}{7} + 1) + 2)\hat{i} + (2\sin(\frac{s}{7} + 1) - 1)\hat{j} + (\frac{s}{7} + 3)\hat{k} \)
(d) \( \vec{r}(t(s)) = (2\cos(s/9))\hat{i} + (2\sin(s/9))\hat{j} + \frac{s}{9}\hat{k} \)
(e) None of the above

(13) A parametrization \( \vec{r}(t) = (f(t), g(t), h(t)) \) of a space curve \( C \) on an interval \( I \) is called smooth if
(a) If \( \vec{r}'(t) \) is continuous, \( \vec{r}''(t) \) is continuous and \( \vec{r}'(t) \) is non-zero for all \( t \in I \).
(b) If \( \vec{r}'(t) \) is non-zero for all \( t \in I \).
(c) If \( \vec{r}'(t) \) is continuous and non-zero for all \( t \in I \).
(d) If \( \vec{r}'(t) \) is continuous for all \( t \in I \).
(e) None of the above

(14) Which of the following statements is false. Assume all the curves below have smooth parametrizations with respect to arc length.
(a) \( \kappa = \left| \frac{dT}{ds} \right| \), where \( \vec{T} \) is a unit tangent vector, is the curvature of the curve \( \vec{r}(t) \) at the point corresponding to \( t \).
(b) \( \kappa = \left| \frac{T'(t)}{|T'(t)|} \right| \), where \( \vec{T} \) is a unit tangent vector, is the curvature of the curve \( \vec{r}(t) \) at the point corresponding to \( t \).
(c) \( \kappa = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'|^3} \) is the curvature of the curve \( \vec{r}(t) \) at the point corresponding to \( t \).
(d) \( \kappa(x) = \frac{|f''(x)|}{[1 + (f'(x)^2)]^{3/2}} \) is the curvature of the graph of \( y = f(x) \) at the point \((x, f(x))\).
(e) All of the formulas can be used to compute the curvature of any space curve \( C \) as described above.

(15) Compute the curvature of \( \vec{r}(t) = (2t, t + 2, 3t + 2) \) at \( t = 4 \).
(a) 0
(b) 1
(c) 2
(d) 3
(e) None of the above

(16) If \( \vec{r}(t) = (3t - 14, \sin t + 9, t^2 + 57) \) is the position of a particle at time \( t \), then its velocity at time \( t \) is equal to the vector
(a) \( \vec{v}(t) = (-3, \cos t, 2t) \)
(b) \( \vec{v}(t) = (3, \cos t, t) \)
(c) \( \vec{v}(t) = (3, \cos^2 t, 2t) \)
(17) Suppose the acceleration of a particle as a function of time $t$ can be computed by the formula $\ddot{a}(t) = (2e^t, 2\cos t, 2t)$ with initial velocity equal to $(0, 0, 0)$ and the initial position being $(0, 0, 0)$. Write down the the formula for the velocity function $\vec{v}(t)$.

(a) $\vec{v}(t) = (2e^t - 2, 2\sin t, t^2)$
(b) $\vec{v}(t) = (e^t - 1, -\cos t + 1, t^2)$
(c) $\vec{v}(t) = (e^t - 1, -2\cos t + 1, \frac{t^2}{2} - 1)$
(d) $\vec{v}(t) = (2e^t + 1, \cos t + 1, t^2)$
(e) None of the above

(18) Suppose the acceleration of a particle as a function of time $t$ can be computed by the formula $\ddot{a}(t) = (3e^t, 3\sin t, 3t)$ with initial velocity equal to $(0, 0, 0)$ and the initial position being $(0, 0, 0)$. Write down the the formula for the position function $\vec{r}(t)$.

(a) $\vec{r}(t) = (e^t - 1, \sin t, t^3)$
(b) $\vec{r}(t) = (3e^t - 1 + t, \cos t + 3, \frac{t^3}{2})$
(c) $\vec{r}(t) = (3e^t - 3t - 3, -3\sin t + 3t, \frac{t^3}{2})$
(d) $\vec{r}(t) = (3e^t + 1 + t, 3\sin t + 1, \frac{t^2}{2})$
(e) None of the above

(19) If $f(x, y, z) = 2xy - z + x^2$, then $f(1, 0, 0)$ is equal to

(a) 0
(b) 1
(c) 2
(d) 3
(e) None of the above

(20) If $f : \mathbb{R}^4 \rightarrow \mathbb{R}$ is a real-valued function of four variables, then its domain consists of

(a) real numbers
(b) pairs of real numbers
(c) triples of real numbers
(d) quadruples of real numbers
(e) None of the above

(21) If $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a function of two variables continuous at $(1,4)$ and $f(1,4) = 2$, then $\lim_{(x,y)\rightarrow(1,4)} f(x,y)$

(a) does not exist.
(b) cannot be determined from these data.
(c) is equal to 2.
(d) is equal to 1.
(e) None of the above

(22) If $f(x, y) = \frac{x^4}{y^4}$, then $\lim_{(x,y)\rightarrow(0,0)} f(x,y)$

(a) is equal to 2.
(b) is equal to 0.
(c) is equal to 3.
(d) does not exist because if we approach along the curve $y = 2x^2$ we get a limit equal to $\frac{1}{2}$, and if we approach along the $y$-axis, we get a limit equal to 0.
(e) None of the above

(23) If \( f(x, y) \) is a function of two variables, then \( \frac{\partial f}{\partial y} \) is defined by the following limit:

(a) \( \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h} \)
(b) \( \lim_{h \to 0} \frac{f(x, y + h) - f(x, y)}{h} \)
(c) \( \lim_{h \to 0} \frac{f(x + h, y + h) - f(x, y)}{h} \)
(d) \( \lim_{h \to 0} \frac{f(x + h, y - h) - f(x, y)}{h} \)
(e) None of the above

(24) If \( f(x, y) = e^{x y^2} + 16y^2 - 17 \), then \( f_x \) is equal to

(a) \( 2xye^{x y^2} \)
(b) \( e^{x y^2} \)
(c) \( y^2 e^{x y^2} \)
(d) \( 2ye^{x y^2} \)
(e) None of the above

(25) If \( f(x, y) = e^{x y^2} + 51x^2 - 31 \), then \( f_y \) is equal to

(a) \( 2xye^{x y^2} \)
(b) \( e^{x y^2} \)
(c) \( 2ye^{x y^2} \)
(d) \( y^2 e^{x y^2} \)
(e) None of the above

(26) If \( f(x, y) = e^{x y^2} + 15y - 7x + 2 \), then \( f_{xx} \) is equal to

(a) \( 2xye^{x y^2} \)
(b) \( e^{x y^2} \)
(c) \( y^4 e^{x y^2} \)
(d) \( y^2 e^{x y^2} \)
(e) None of the above

(27) If \( f(x, y) = e^{x y^2} + 25x - 3y - 3 \), then \( f_{yy} \) is equal to

(a) \( 2xe^{x y^2} + 4x^2 y^2 e^{x y^2} \)
(b) \( e^{x y^2} \)
(c) \( y^4 e^{x y^2} \)
(d) \( y^2 e^{x y^2} \)
(e) None of the above

(28) If \( f(x, y) = e^{x y^2} + 13x^2 + 3y^4 + 12 \), then \( f_{xy} \) is equal to

(a) \( e^{x y^2} 2x^3 \)
(b) \( e^{y^4} e^{x y^2} + e^{x y^2} 2x^3 \)
(c) \( 2y e^{x y^2} + 2x y^3 e^{x y^2} \)
(d) \( y^2 e^{x y^2} - e^{x y^2} 2x^3 \)
(e) None of the above

(29) If \( f(x, y) = e^{x y^2} - 17x^3 + x + 2y^2 + 10 \), then \( f_{yx} \) is equal to

(a) \( 2xye^{x y^2} + e^{x y^2} 2x^3 \)
(b) \( e^{x y^2} + e^{x y^2} 2x^3 \)
(c) $2ye^{xy^2} + 2xy^3e^{xy^2}$
(d) $y^2e^{xy^2} - e^{xy^2}2x^3$
(e) None of the above

(30) If $f$ is any function from $\mathbb{R}^2$ to $\mathbb{R}$ defined on an open (without boundary) disk $D \subset \mathbb{R}^2$, $(a, b) \in D$, and $f_{xy}$ and $f_{yx}$ are both defined and continuous on $D$, then

(a) $f_{xy}(a, b) = -f_{yx}(a, b)$
(b) $f_x(a, b) = f_y(a, b)$
(c) $f_{xy}(a, b) = f_{yx}(a, b)$
(d) $f_{xy}(a, b) = -f_x(a, b)$
(e) None of the above

(31) Write an equation of a plane passing through the points $(1, 2, 3), (2, 7, 3), (4, 3, 3)$

(a) $z = 3$
(b) $3x + 2y - 7z = 12$
(c) $y = 2$
(d) $x = 2$
(e) None of the above

(32) Let $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ be three non-collinear points. Let $\vec{u} = (x_1 - x_2, y_1 - y_2, z_1 - z_2), \vec{v} = (x_1 - x_3, y_1 - y_3, z_1 - z_3), (a, b, c) = \vec{u} \times \vec{v}$. In this case which of the equations below is the equation of the plane passing through $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$?

(a) $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
(b) $a(x - x_2) + b(y - y_2) + c(z - z_2) = 0$
(c) $a(x - x_3) + b(y - y_3) + c(z - z_3) = 0$
(d) All of the above
(e) None of the above

(33) If $\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k},$ where $f(t), g(t), h(t)$ are differentiable functions, then $\vec{r}'(t)$

(a) is a vector function with three components
(b) is not a vector function.
(c) is a vector function with two components.
(d) is a vector function with four components.
(e) None of the above

(34) Compute the curvature of $\vec{r}(t) = (3 \cos t, 3 \sin t, 1)$ at $t = 3$.

(a) $\frac{1}{3}$
(b) $\frac{1}{2}$
(c) $2$
(d) $3$
(e) None of the above

(35) Write an equation of the line passing through points $(1, 2, 2)$ and $(3, 4, 5)$.

(a) $x = 1 + t, y = 2 + 2t, z = 2 + 3t$
(b) $x = 1 - t, y = 1 + 2t, z = 1 + 3t$
(c) $x = 2 + t, y = 1 + 2t, z = 3 + 3t$
(d) $x = 1 + t, y = 3 + t, z = 1 + t$
(e) None of the above

(36) Write an equation of the plane perpendicular to vector \((1, 2, 3)\) and passing through the point \((4, 5, 6)\).
(a) \((x - 4) + 2(y - 5) + 3(z + 6) = 0\)
(b) \((x - 4) + 2(y - 5) + 3(z - 6) = 1\)
(c) \((x - 4) - 2(y - 5) + 3(z - 6) = 0\)
(d) \((x - 4) + 2(y - 5) + 3(z - 6) = 0\)
(e) None of the above

(37) Find an equation of the tangent plane to \(z = x^2 + y^2\) at \((1,2)\).
(a) \(z - 5 = 2(x + 1) + 4(y + 2)\)
(b) \(z - 5 = 4(x - 1) + 2(y - 2)\)
(c) \(z + 5 = 2(x - 1) + 4(y - 2)\)
(d) \(z - 5 = 2(x - 1) + 4(y - 2)\)
(e) None of the above

(38) Let \(z = x^2 + y^3, x(t) = (3t + \sin t), y(t) = 5t\) and compute \(dz/dt\).
(a) \(dz/dt = (3 + \sin t) + 15y^2\)
(b) \(dz/dt\) does not exist
(c) \(dz/dt = 2x + 15y^2\)
(d) \(dz/dt = 2x(3 + \cos t) + 15y^2\)
(e) None of the above

(39) Let \(z = x^2 + y^3, x(t) = (3s + \sin t), y(s) = 5s + t\) and compute \(\partial z / \partial s\).
(a) \(\partial z / \partial s = (3 + \sin t) + 15y^2\)
(b) \(\partial z / \partial s = 6xt + 15y^2\)
(c) \(\partial z / \partial s\) does not exist
(d) \(\partial z / \partial s = 6x + 15y^2\)
(e) None of the above