ERRATUM TO: "GEOMETRIC MODELS FOR THE SPECTRA OF CERTAIN GELFAND PAIRS ASSOCIATED WITH HEISENBERG GROUPS"

CHAL BENSON AND GAIL RATCLIFF

The paper contains an error which necessitates some revisions to the proof of our main result, Theorem 1.2. In fact the polynomial functions $\widehat{D}_\alpha \in \mathbb{C}[\mathfrak{a}^*]$ ($\alpha \in \Lambda$), discussed in Section 2.2, need not, in general, be invariant under the little Weyl group $W_\circ$ as stated. One needs to introduce a "ϱ-shift" (half the sum of the positive roots) to achieve $W_\circ$-invariance. This means that although $\mathrm{top}(\widehat{D}_\alpha) \in \mathbb{C}[\mathfrak{a}^*]^{W_\circ}$ the polynomial $\widehat{D}_\alpha$ itself need not lie in the image of the mapping $\bar{\rho}$ given in Equations 2.1 and 2.3. Thus we cannot define polynomials $E_\alpha \in \mathbb{C}[V_\mathbb{R}]^K$ as in Definition 2.1 or obtain the related functions $\varepsilon_L \in C(\mathfrak{h}_V^*)^K$ as claimed in Proposition 4.2.

Lemma A.2 below provides a technical tool needed to revise the proof for Theorem 1.2. First we require the following substitute for Lemma 2.7 from the paper.

Lemma A.1. *For a well-behaved multiplicity free action $K : V$ and $\alpha, \beta \in \Lambda$ one has*

$$\big(top(\widehat{D}_\alpha)\big)(\beta) = p_\alpha(v_\beta).$$

Proof. Equation 2.3 yields

$$\bar{\rho}(p_\alpha)(\beta) = \widetilde{p}_\alpha\big(z, \eta(z,\beta)\big)$$

for any $z \in V$ satisfying $h_\beta(z) \neq 0$. Condition (i) in Definition 2.4 allows us to take $z = v_\beta$ and condition (ii) gives $\eta(v_\beta, \beta) = \langle \cdot, v_\beta \rangle = v_\beta^*$. So then

$$\big(\mathrm{top}(\widehat{D}_\alpha)\big)(\beta) = \bar{\rho}(p_\alpha)(\beta) = \widetilde{p}_\alpha(v_\beta, v_\beta^*) = p_\alpha(v_\beta). \quad \square$$

Lemma A.2. *Let $K : V$ be a well-behaved multiplicity free action, (β_n) a sequence in Λ and (λ_n) a sequences in $\mathbb{R}^\times$ with $\lim \lambda_n = 0$ and $(|\lambda_n|\beta_n)$ converging in $\mathfrak{a}^*$. Then $\lim \widehat{L_\alpha}(\phi_{\beta_n,\lambda_n}) = \lim p_\alpha(\sqrt{2|\lambda_n|}\, v_{\beta_n})$ for all $\alpha \in \Lambda$.*

Proof. Recall that $\widehat{L_\alpha}(\phi_{\beta_n,\lambda_n}) = (2|\lambda_n|)^{|\alpha|}\widehat{D}_\alpha(\beta_n)$ by Equation 3.1. So now

$$\lim \widehat{L_\alpha}(\phi_{\beta_n,\lambda_n}) = \lim (2|\lambda_n|)^{|\alpha|}\widehat{D}_\alpha(\beta_n) = \lim (2|\lambda_n|)^{|\alpha|}\widehat{D}_\alpha((2|\lambda_n|)^{-1}\beta) = \big(\mathrm{top}(\widehat{D}_\alpha)\big)(\beta)$$

where $\beta := \lim(2|\lambda_n|\beta_n)$. On the other hand, using Lemma A.1,

$$\begin{aligned}\big(\mathrm{top}(\widehat{D}_\alpha)\big)(\beta) &= \lim\big(\mathrm{top}(\widehat{D}_\alpha)\big)(2|\lambda_n|\beta_n) \\ &= \lim(2|\lambda_n|)^{|\alpha|}\big(\mathrm{top}(\widehat{D}_\alpha)\big)(\beta_n) \\ &= \lim(2|\lambda_n|)^{|\alpha|} p_\alpha(v_{\beta_n}) = \lim p_\alpha(\sqrt{2|\lambda_n|}v_{\beta_n}). \qquad \square\end{aligned}$$

Revised Proof of Theorem 1.2. Assume that the multiplicity free action $K : V$ is well-behaved. Let $(\varphi_n)_{n=1}^{\infty}$ be a sequence in $\Delta(K, H_V)$, $\varphi \in \Delta(K, H_V)$, and write $\mathcal{K}_n = \Psi(\varphi_n)$, $\mathcal{K} = \Psi(\varphi)$. We will show that

$$\varphi_n \longrightarrow \varphi \text{ in } \Delta(K, H_V) \iff \mathcal{K}_n \longrightarrow \mathcal{K} \text{ in } \mathfrak{h}_V^*/K.$$

The Heisenberg fan model shows that $\varphi_n \longrightarrow \varphi$ if and only if $\widehat{L}(\varphi_n) \longrightarrow \widehat{L}(\varphi)$ for every $L \in \mathbb{D}_K(H_V)$. By considering subsequences we may assume that either every φ_n is a spherical function of Type 2 or every φ_n is of Type 1.

($\Rightarrow$): First assume that $\varphi_n \longrightarrow \varphi$. We will show $\mathcal{K}_n \longrightarrow \mathcal{K}$.

Case 1: Suppose that each φ_n is a spherical function of Type 2. For some points $w_n \in V$ one has $\varphi_n = \eta_{K\cdot w_n}$ and $\mathcal{K}_n = (K \cdot w_n) \times \{0\}$. As $\widehat{T}\varphi = \lim \widehat{T}\varphi_n = 0$ it follows that φ is also of Type 2. So now $\varphi = \eta_{K\cdot w}$ and $\mathcal{K} = (K \cdot w) \times \{0\}$ for some $w \in V$. As $\widehat{L_\circ}(\varphi_n) = |w_n|^2$ (see (4.2)) converges to $\widehat{L_\circ}(\varphi) = |w|^2$ it follows that (w_n) is a bounded sequence. Passing to a subsequence we may assume that (w_n) converges in V, with $\lim w_n = w'$ say. Now for each $\alpha \in \Lambda$ we observe that

$$p_\alpha(w') = \lim p_\alpha(w_n) = \lim \widehat{L_\alpha}(\varphi_n) = \widehat{L_\alpha}(\varphi) = p_\alpha(w).$$

As $\{p_\alpha \; : \; \alpha \in \Lambda\}$ is a basis for $\mathbb{C}[V_\mathbb{R}]^K$ and the invariants for a compact linear action separate orbits it follows that $K \cdot w' = K \cdot w$. Hence $\mathcal{K}_n$ converges to $\mathcal{K}$ in $\mathfrak{h}_V^*/K$.

Case 2: Suppose that each φ_n is a spherical function of Type 1,

$$\varphi_n = \phi_{\beta_n,\lambda_n} \quad \text{and} \quad \mathcal{K}_n = \sqrt{2|\lambda_n|}\,(K \cdot v_{\beta_n}) \times \{\lambda_n\}$$

say. Let

$$\lambda := \widehat{T}\varphi = \lim \widehat{T}\varphi_n = \lim \lambda_n.$$

Case 2(a): If $\lambda \neq 0$ then $\varphi = \phi_{\beta,\lambda}$ and $\mathcal{K} = \sqrt{2|\lambda|}\,(K \cdot v_\beta) \times \{\lambda\}$ for some $\beta \in \Lambda$. As $\widehat{L_\circ}(\varphi_n) = 2|\lambda_n||\beta_n|$ converges to $\widehat{L_\circ}(\varphi) = 2|\lambda||\beta|$, it follows that $\lim |\beta_n| = |\beta|$. As $\{\alpha \in \Lambda \; : \; |\alpha| = |\beta|\}$ is a finite set we can assume, by passing to a subsequence, that $\beta_n = \beta$ for every n. So now $\mathcal{K}_n = \sqrt{2|\lambda_n|}\,(K \cdot v_\beta) \times \{\lambda_n\}$ with $\lambda_n \longrightarrow \lambda$ and thus $\mathcal{K}_n \longrightarrow \mathcal{K}$ as desired.

Case 2(b): If $\lambda = 0$ then $\varphi = \eta_{K\cdot w}$ and $\mathcal{K} = (K \cdot w) \times \{0\}$ for some $w \in V$. Moreover $\widehat{L_\circ}(\varphi_n) = 2|\lambda_n||\beta_n| = 2|\lambda_n||v_{\beta_n}|^2$ converges to $\widehat{L_\circ}(\varphi) = |w|^2$ and thus $\sqrt{2|\lambda_n|}\,v_{\beta_n}$ is a bounded sequence. By passing to a subsequence we may assume this converges in V and write $v := \lim \sqrt{2|\lambda_n|}\,v_{\beta_n}$. Applying the moment map it follows that $2|\lambda_n|\beta_n$ converges to a point $\beta \in \mathfrak{a}^*$ with $\beta_\mathfrak{k} = \tau(v)$. Now Lemma A.2 yields

$$p_\alpha(w) = \widehat{L_\alpha}(\varphi) = \lim \widehat{L_\alpha}(\phi_{\beta_n,\lambda_n}) = \lim p_\alpha\big(\sqrt{2|\lambda|}\,v_{\beta_n}\big) = p_\alpha(v)$$

for each $\alpha \in \Lambda$. As in Case 1 this implies that $K \cdot w = K \cdot v$ and thus $\mathcal{K} = \lim \mathcal{K}_n$.

($\Leftarrow$): Next assume conversely that $\mathcal{K}_n \longrightarrow \mathcal{K}$. We will show $\varphi_n \longrightarrow \varphi$.

Case 1: Suppose that each φ_n is a spherical function of Type 2. Hence $\mathcal{K}_n \subset V \times \{0\}$ for all n and as $\mathcal{K}_n \to \mathcal{K}$ it follows that $\mathcal{K} \subset V \times \{0\}$ and that φ is of Type 2. So

$$\varphi_n = \eta_{K\cdot w_n},\ \mathcal{K}_n = (K \cdot w_n) \times \{0\}; \qquad \varphi = \eta_{K\cdot w},\ \mathcal{K} = (K \cdot w) \times \{0\}$$

say. Now $K \cdot w_n \longrightarrow K \cdot w$ and hence $\big(\widehat{L_\alpha}(\varphi_n) = p_\alpha(w_n)\big) \longrightarrow \big(p_\alpha(w) = \widehat{L_\alpha}(\varphi)\big)$ for each $\alpha \in \Lambda$. It follows that $\varphi_n \longrightarrow \varphi$.

Case 2: Suppose that each φ_n is a spherical function of Type 1, and write

$$\varphi_n = \phi_{\beta_n,\lambda_n} \quad \text{and} \quad \mathcal{K}_n = \sqrt{2|\lambda_n|}\,(K \cdot v_{\beta_n}) \times \{\lambda_n\}.$$

As $(\mathcal{K}_n)$ converges so does (λ_n). Let $\lambda := \lim \lambda_n$.

Case 2(a): If $\lambda \neq 0$ then $\varphi = \phi_{\beta,\lambda}$ and $\mathcal{K} = \sqrt{2|\lambda|}\,(K \cdot v_\beta) \times \{\lambda\}$ for some $\beta \in \Lambda$ with $K \cdot v_{\beta_n} \longrightarrow K \cdot v_\beta$. As Λ is a discrete set it follows that the sequence (β_n) is eventually constant. Thus $\varphi = \phi_{\beta,\lambda} = \lim \phi_{\beta,\lambda_n} = \lim \varphi_n$.

Case 2(b): If $\lambda = 0$ then $\varphi = \eta_{K\cdot w}$ and $\mathcal{K} = (K \cdot w) \times \{0\}$ for some $w \in V$. As $\mathcal{K}_n \longrightarrow \mathcal{K}$ we have $\sqrt{2|\lambda_n|}\,(K \cdot v_{\beta_n}) \longrightarrow K \cdot w$ and by passing to a subsequence we may assume that $\sqrt{2|\lambda_n|}\,v_{\beta_n}$ converges to a point $v \in K \cdot w$. Applying the moment map it follows that $2|\lambda_n|\beta_n$ converges in $\mathfrak{a}^*$. Again using Lemma A.2 we obtain

$$\lim \widehat{L_\alpha}(\varphi_n) = \lim p_\alpha\big(\sqrt{2|\lambda_n|}\,v_{\beta_n}\big) = p_\alpha(v) = p_\alpha(w) = \widehat{L_\alpha}(\varphi)$$

for each $\alpha \in \Lambda$ and hence $\varphi_n \longrightarrow \varphi$ as claimed. □

Finally we note that without the functions ε_L given by Proposition 4.2 we do not obtain an explicit homeomorphism between the orbital and Heisenberg fan models for $\Delta(K, H_V)$, as claimed in Corollary 4.4.

Dept of Mathematics, East Carolina University, Greenville, NC 27858
E-mail address: `bensonf@ecu.edu, ratcliffg@ecu.edu`